Proposition 1

If There be any number of magnitudes whatever which are, respectively, equimultiples of many magnitudes equal in multitude, then, whatever multiple one of the magnitudes is of one, that multiple will also be of all

given

magnitudes AB, CD be equimultiples of E, F in equal multitude

required

whatever multiple AB is of E, that multiple will AB, CD also be of E, F

AB is the same multiple of E that CD of F

divide AB into magnitudes AG, GB
AG, GB = E
divide CD into magnitudes CH, HD
CH, HD = F

AG, GB = CH, HD

AG = E
CH = F
AG, CH = E, F

For the same reason,
GB = E
GB, HD = E, F

as many magnitudes there are in AB = E, so many also are in AB, CD = E, F
whatever multiple AB is of E, that multiple will AB, CD be of E, F.

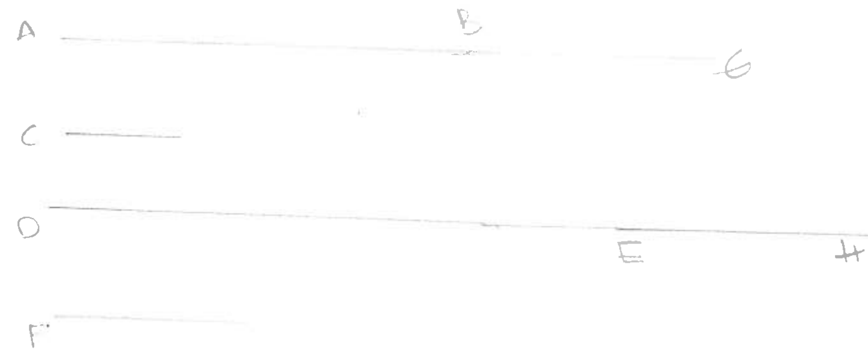
Q.E.D.

Proposition 2

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of a second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth.

given

first magnitude AB be the same multiple of a second C, that a third DE is of a fourth F. and a fifth BG, be the same multiple as the second, C, that a sixth, EH is of fourth F



required

The sum of the first and fifth AG will be the same multiple of the second, C, that the sum of third and sixth, DH, is of fourth, F.

AB is the same multiple of C that DE of F

as many magnitudes there are in AB = C, there also are in DE = F

For the same reason,

as many as there are in BG = C there also are in EH = F

as many as there are in AG = C There also are in DH = F

whatever multiple AG is of C DH is of F

The sum of the first and fifth, AG, is the same multiple of the second, C, that the sum of the third and sixth, DH, is of the fourth, F

Q.E.D.

Proposition 3

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also ex aequali the magnitudes taken will be equimultiples respectively, the one of the second and the other of the fourth.

given:

first magnitude A be the same multiple of a second B, that a third C is of fourth D.
equimultiples EF, GH be taken of A, C.

required

EF is the same multiple of B that GH is of D

EF is the same multiple of A that GH of C

as many magnitudes there are in EF equal to A there are in GH equal to C

divide EF in EK, KF = A
divide GH in GL, LH = C
magnitudes EK, KF = magnitude GL, LH

A is the same multiple of B than C of D
EK = A
GL = C

EK is the same multiple of B tan GL of D

first magnitude EK is the same multiple of a second B that a third GL is of fourth D and a fifth KF is also multiple of second B, than a sixth LH is of the fourth D. [V.2]

Q.E.D

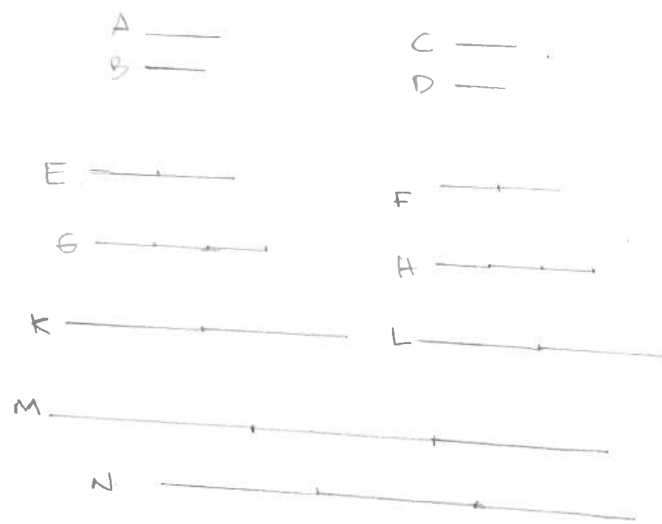
Proposition 4

If a first magnitude have to a second magnitude the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.

given:

first magnitude A have to the second, B, the same ratio as a third C to a fourth D

equimultiples E, F be taken of A, C and G, H of B, D.



required.

E is to G so is F to H

Equimultiples K, L be taken of E, F
Equimultiples M, N be taken of G, H

E is the same multiple A that F of C
equimultiples K, L of E, F have been taken
K is the same multiple A that L of C [V.3]

For the same reason,
M is the same multiple of B that N of D.

A is to B, so is C to D
A, C equimultiples of K, L
B, D equimultiples M, N

K is in excess of M,
L is in excess of N
If it is equal, equal
If it is less, less [V. Def. 5]

K, L are equimultiples of E, F
M, N are equimultiples of G, H.

E is to G
so is
F to H [V. Def. 5]

Q.E.D.

Proposition 5

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole

given

magnitude AB = magnitude CD that the part AE subtracted is of the part CF subtracted

required

remainder EB = remainder FD that the whole AB is of the whole CD

whatever multiple AE is of CF let EB be made that multiple of CG

AE is the same multiple of CF that EB of CD

AE is the same multiple of CF that AB of GF [V.1]

→Assumption

AE is the same multiple of CF that AB of CD

AB is the same multiple of GF, CD

GF = CD

→Subtract CF

GC = FD

AE is the same multiple of CF that EB of GC

GC = FD

AE is the same multiple of CF that EB of FD

→hypothesis.

AE is the same multiple of CF that AB of CD

EB is the same multiple of FD that AB of CD

The remainder EB will be the same multiple of the remainder FD that the whole AB is of the whole CD.

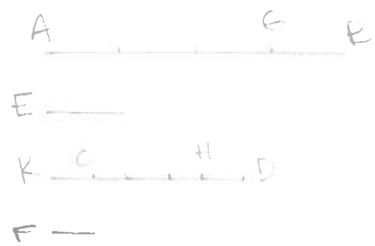
Q.E.D.

Proposition 6

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders also are either equal to the same or equimultiples of them.

given.

magnitudes AB, CD be equimultiples of magnitudes E, F.
let AG, CH subtracted from the equimultiples of the same two E, F



required:

remainders GB, HD are either equal to E, F or equimultiples of them.

GB = E
→ HD = F

CK = F

AG is the same multiple of E that CH of F
GB = E   KC = F
AB is the same multiple of E that KH of F   [V. 2]

→ hypothesis
AB is the same multiple of E that CD of F
KH is the same multiple of F that CD of F

magnitudes KH = CD

→ subtract CH

KC = HD

F = KC
HD = F

GB = E
HD = F

similarly,
GB be a multiple of E
HD is the same multiple of F

Q.E.D.

Proposition 7

Equal magnitudes have to the same ratio, as also has the same to equal magnitudes.

given

A, B be equal magnitudes
C any other magnitude

required

magnitudes A, B have the same ratio to C and C has the same ratio as A, B.

equimultiples D, E of A, B
equimultiple F of C

D is the same multiple of A that E of B

A = B
D = E

→ F is another magnitude

D is in excess of F,
E is in excess of F
If equal to it, equal
and, if less, less

D, E are equimultiples of A, B
F is multiple of C
A is to C
AS B is to C [V. Def. 5]

→ C has the same ratio to A, B

with the same construction,
D = E
F another magnitude

F is in excess of D, it is also in excess of E.
If equal, equal
If less, less

F is a multiple of C
D, E are multiples of A, B

C is to A
AS C is to B [V. Def. 5]

Q.E.D.

PORISM:

From this it is manifest that, if any magnitudes are proportional, they will also be proportional inversely. [V. Def. 13]

Proposition 8

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater.

given:

AB, C be unequal magnitudes

AB > C

D another magnitude

required:

AB has to D a greater ratio than C has to D, and D has a greater ratio than it has to AB.

CASE 1:

AE < EB

AE multiplied

FG multiple > D

whatever multiple FG is of AE
GH is the same multiple of EB
K of C

L double D

M triple D

N quadruple of D
first multiple > K

K < N (first)
K > M

FG same multiple of AE
that GH of EB
FH is the same multiple of AE
that FH of AB [V.1]

FG is the same multiple of AE
that K of C

FH is the same multiple of AB
that K of C

FH, K are equimultiples of AB, C

→ GH is the same multiple of EB
that K of C

EB = C
GH = K

K > M
GH > M

FG > D

FH > D, M

D + M = N
M is triple D
M, D are quadruple D
N = quadruple D
M, D = N

FH>M,D

FH is in excess of N

K is not an excess of N

FH, K are equimultiples of AB, C, while N is multiple of D
AB has to D a greater ratio than C has to D [V.Def.7]

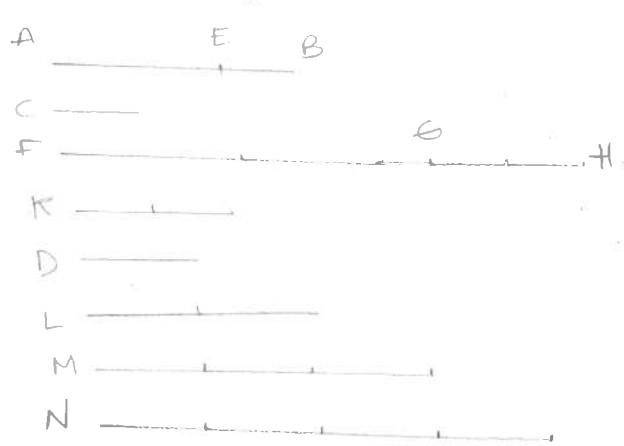
→ D also has to C a greater ratio than D has to AB

with the same construction,
N is in excess of K
N is not in excess of FH

N is a multiple of D, FH, K are equimultiples of AB, C.
D has a greater ratio to C than D to AB [V.Def.7]

CASE 2:

AE>EB



EB if multiplied will sometime be greater than D [V.Def.4]

GH multiple of EB
GH>D

whatever multiple GH is of EB
FG the same multiple of AE
K of C

Similarly,

FH, K are equimultiples of AB, C
Similarly,

N multiple of D, the first greater than FG
FG>M

GH>D
FH is in excess of D, M (N)

K is not in excess of N, inasmuch as FG also, which is greater than GH
That is, than K, is not in excess of N

Q.E.D

Proposition 9

magnitudes which have the same ratio to the same are equal to one another; and magnitudes to which have the same ratio are unequal.

given

A, B have the same ratio to C

required.

A = B

otherwise,
each of these magnitudes would not have had the same ratio to C [V.8]

but they do,

A = B

let C have the same ratio to A, B

A = B

otherwise,
C would not have had the same ratio of A, B [V.8]

But it has,

A = B

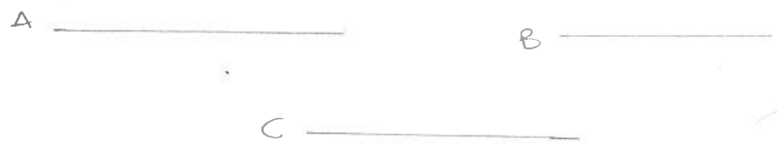
Q.E.D.

Proposition 10

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

<u>given:</u>

A has a greater ratio to C than B has to C



<u>required:</u>

A>B

If not A is equal or less to B

→ A is not equal to B
for in that case A,B would have had the same ratio to C, [V.7]
but they don't

A ≠ B

→ A is not less than B
for in that case A would have had to C a less ratio than B to C
but it has not

A is not less than B

But they are not equal

A>B

→ C has to B a greater ratio than C to A
B<A

If not it is equal or greater

→ B is not equal to A
For in that case, C would have had the same ratio to A,B [V.7]
but it has not

B is not equal to A

→ B is not greater than A
For if in that case, C would have had to B a less ratio than it has to A [V.8]
but it has not

B is not greater than A

But they are not equal

B<A

Q.E.D

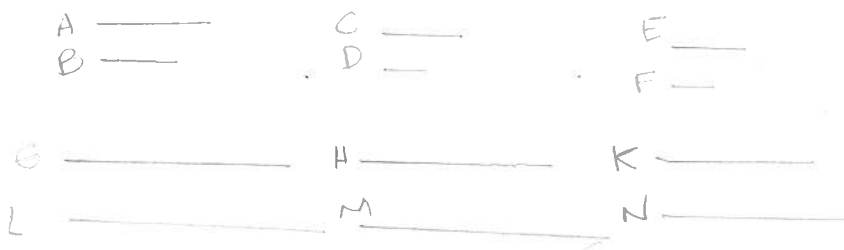
Proposition 11

ratios which are the same with the same ratio are also the same with one another

given

as A is to B
C is to D

as C is to D
E is to F



required

as A is to B
E is to F

G, H, K equimultiples of A, C, E
L, M, N equimultiples of B, D, F

as A is to B
C is to D
of A, C equimultiples G, H
of B, D equimultiples L, M
If G is in excess of L
H is in excess of M
If equal, equal
If less, less

as C is to D
E is to F
of C, E equimultiples H, K.
of D, F equimultiples M, N
If H is in excess of M
K is in excess of N
If equal, equal
If less, less

H is in excess of M
G is in excess of L

If G is in excess of L
K is in excess of N
If equal, equal
If less, less

G, K are equimultiples of A, E
L, N are equimultiples of B, F

as A is to B
E is to F

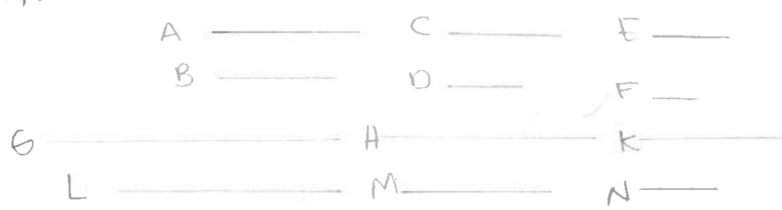
QED

Proposition 12

If any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so will all the antecedents be to all consequents.

given

magnitudes A, B, C, D, E, F
are proportional
as A is to B
C is to D
E is to F



required

as A is to B
A, C, E is to B, D, F

G, H, K equimultiples of A, C, E
L, M, N equimultiples of B, D, F

as A is to B
C is to D
E is to F

of A, C, E equimultiples G, H, K
of B, D, F equimultiples L, M, N

If G is in excess of L.
H is in excess of M
K of N
If equal, equal
If less, less.

→ in addition
If G is in excess of L
G, H, K are in excess of L, M, N

G and G, H, K are equimultiples of
A, A, C, E.

since, if any number of magnitudes whatever are respectively equimultiples of any magnitudes equal in multitude, whatever multiple one of the magnitudes is of one, that multiple will be of all [V. 1]

For the same reason,

L and L, M, N are
equimultiples of B, B, D, F

As A is to B
A, C, E is to B, D, F
[V. def. 5]

Q.E.D

Proposition 13

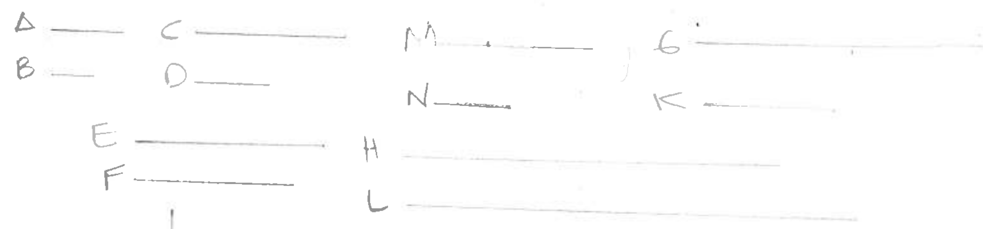
If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth.

given:

magnitude A have to
second, B, the same
atio as a third C has
to the fourth D;
he third C has to the
ourth, D a greater ratio
han a fifth E has to
a sixth F.

equired:

e first A has to
he second B, a
reater ratio than
e fifth E to the
xth F.



here are equimultiples of
C,E and D,F and other
equimultiples such that
C is in excess of D
while multiple E is not in
excess of F [V.Def 7]

G,H equimultiples of C,E
K,L equimultiples of D,F
G is in excess of K
H is not in excess of L

whatever multiple G is of C
M is of A

whatever multiple K is of D
N is of B

As A is to B
C is to D

M,G equimultiples of A,C
N,K equimultiples of B,D
If M is in excess N
G is in excess of K
If equal, equal
If less, less [V.Def 5]

G is in excess of K
M is in excess of N

H is not in excess of L
M,H are equimultiples of A,E
N,L equimultiples of B,F

A has to B a greater
ratio than E to F [V.Def 7]

QED.

Proposition 14

If a first magnitude have to a second the same ratio as a third to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less

Given

first magnitude A have The same ratio to a second B as a third C has to a fourth D.

$A > C$

required:

$B > D$

$A > C$

B is another magnitude

A has to B a greater ratio than C to B

as A is to B
C is to C

C has a greater ratio to D than C to B [V.13]

But that to which the same has a greater ratio is less; [V.10]

$D < B$

$B > D$

Similarly,

If $A = C$

$B = D$

and

If $A < C$

$B < D$

Q.E.D.

Proposition 15

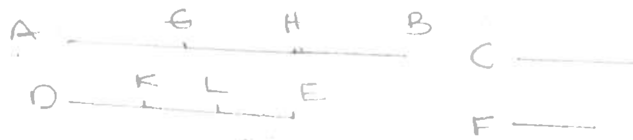
Parts have the same ratio as the same multiples of them taken in corresponding order.

given:

AB the same multiple of C that DE is of F

required:

as C is to F
AB is to DE



AB is the same multiple of C that DE is of F

as many magnitudes as there are in AB = C, there are in DE = F

AB divided into AG, GH, HB = C
DE divided into DK, KL, LE = F

multitude AG, GH, HB = multitude DK, KL, LE

AG = GH = HB
DK = KL = LE

as AG is to DK
GH is to KL
HB is to LE [V.7]

as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents [V.12]

as AG is to DK
AB is to DE

AG = C
DK = F

as C is to F
AB is to DE

Q.E.D.

Proposition 16

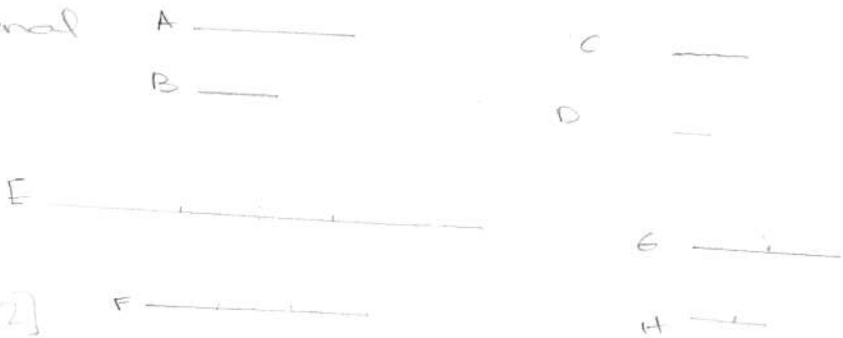
If four magnitudes be proportional, they will also be Proportional alternately.

Given:

A, B, C, D 4 proportional magnitudes
as A is to B
C is to D

Required

They will also be so alternately [V. Def. 12]
as A is to C
B is to D



E, F equimultiples of A, B
G, H equimultiples of C, D

E is the same multiple of A
that F is of B
Parts have the same ratio
as the same multiples of them [V.15]
as A is to B
E is to F

As A is to B
C is to D
as C is to D.
E is to F [V.11]

G, H equimultiples of C, D
as C is to D
G is to H [V.15]

As C is to D
E is to F

as E is to F [V.11]
G is to H

If four magnitudes be Proportional, and the first be greater than the third, The second will also be greater than the fourth.
If equal, equal
If less, less [V.14]

If E is in excess of G
F is in excess of H.

E, F equimultiples of A, B
G, H equimultiples of C, D
as A is to C
B is to D [V. Def. 5]

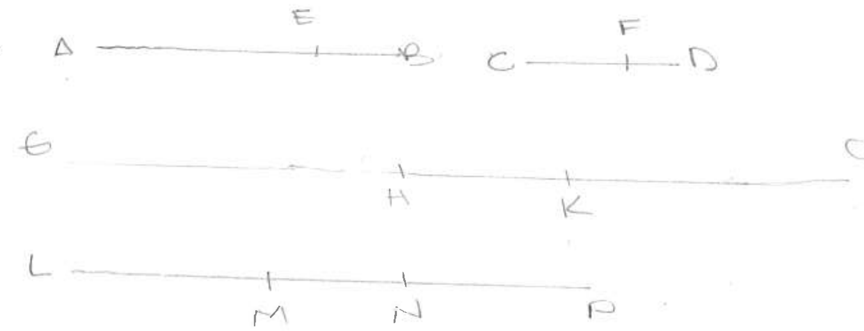
QED

Proposition 17

If magnitudes be proportional componendo they will also be proportional separando

given:

AB, BE, CD, DF proportional componendo [V.def 14]
as AB is to BE
CD is to DF



required

They will also be proportional separando [V. Def. 15]
as AE is to EB
CF is to DF

GH, HK, LM, MN equimultiples of AE, EB, CF, FD

KO, OP equimultiples of EB, FD

GH is the same multiple of AE
That HK of EB
GH is the same multiple of AE
That GK of AB [V.1]

GH is the same multiple of AE
That LM of CF
GK is the same multiple of AB
that LM of CF.

LM is the same multiple of CF
that MN of FD
LM is the same multiple of CF
that LN of CD [V.1]

LM same multiple of CF
that GK of AB

GK is the same multiple of AB
that LN of CD

GK, LN are equimultiples of AB, CD

HK is the same multiple of EB
That MN of FD
KO is the same multiple of EB
That NP of FD

HO is the same multiple of EB
That MP of FD [V.2]

AS AB IS TO BE
CD IS TO DF

GK, LN equimultiples of AB, CD
HO, MP equimultiples of EB, FD

GK is in excess of HO
LN is in excess of MP

GK in excess of HO
→ subtract HK
GH in excess of KO

If GK is in excess of HO
LN is in excess of MP
→ subtract MN
LM is in excess of NP

GH is in excess of KO
LM is in excess of NP

Similarly,

if GH = KO

LM = NP

if less, less

GH, LM equimultiples of AE, CF

KO, NP equimultiples of EB, FD

as AE is to EB

CF is to FD

Q.E.D

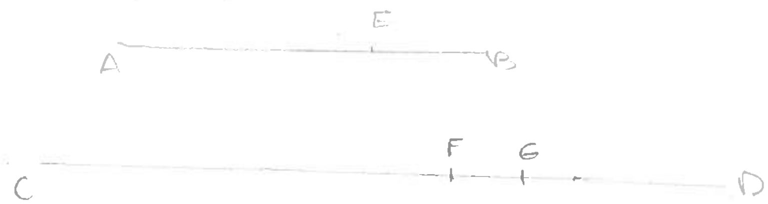
Proposition 18

If magnitudes be proportional separando, they will be proportional componendo.

given:

AE, EB, CF, FD proportional separando [V. def 15]

as AE is to EB
CF is to FD



required

they will also be proportional componendo [V. def 14]

as AB is to BE
CF is to FD

If CD be not to DF as AB to BE
Then as AB is to BE
CD will either be less than DF or greater.

→ a ratio to a less magnitude DG
as AB is to BE
CD is to DG

They are proportional componendo
They will also be proportional separando [V.17]

as AE is to EB
CG is to GD

→ hypothesis
as AE is to EB
CF is to FD

as CG is to GD
CF is to FD [V.11]

First CG is greater than third CF
second GD is greater than fourth FD [V.14]

But it is also less:
IMPOSSIBLE

As AB is to BE
so is not CD to a less magnitude than FD

Similarly,
neither it is in that ratio to a greater;
It is in that ratio to FD

Q.E.D.

Proposition 19

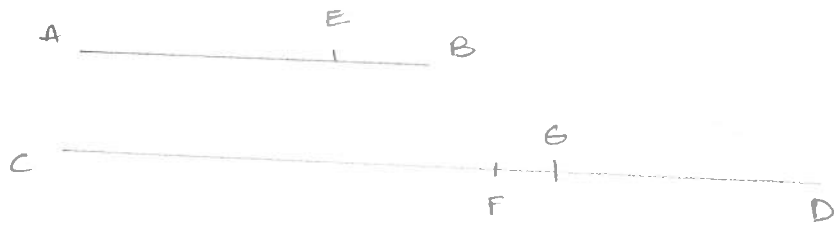
If, as a whole is to a whole, so is a part subtracted to a part, remainder will also be to the remainder as whole to whole.

given:

whole AB is to the whole CD,
part AE subtracted to the part CF subtracted

required:

remainder EB will be to the remainder FD as the whole AB to the whole CD



As AB is to CD
AE is to CF

alternately,
as BA is to AE
CD is to CF [V.16]

the magnitudes are proportional componendo and separando [V.17]

as BE is to EA
DF is to CF
alternately,
BE is to DF
EA is to FC [V.16]

*hypothesis
as AE is to CF
whole AB is to whole CD

The remainder EB will be to the remainder FD
as the whole AB is to the whole CD
[V.11]

QED.

PORISM:

From this it is manifest that, if magnitudes be proportional componendo [V.def.14], they will also be proportional convertendo [V.def.16]

Proposition 20

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if ex aequali the first be greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less.

given:

magnitudes A, B, C
equal to D, E, F in
multitude.
Taken 2 and 2 are in
The same ratio.
AS A IS TO B
D IS TO E
and, AS B IS TO C
E IS TO F

A > C ex aequali [V, def. 17]

required:

D > F
If A = C equal,
If less, less

A > C
B other magnitude
The greater has to the same a greater ratio than the less has [V. 8]
A has to B a greater ratio than C to B

AS A IS TO B
D IS TO E

AS C IS TO B
F IS TO E

D has to E a greater ratio than C to B

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater [V. 10]

D > F

similarly,
If A = C
D = F
and if less, less

QED

Proposition 21

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, then, if ex aequali the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less.

given:

magnitudes A, B, C and D, E, F equal in multitude.
taken two and two are in the same ratio.
their proportion is perturbed [V. def. 18]

as A is to B
E is to F

and as B is to C
D is to E

A > C ex aequali [V. def 17]

required:

D > F

If A = C, equal
If less, less

A > C
B other magnitude

A has to B a greater ratio than C to B [V. 8]

As A is to B
E is to F

and as C is to B
E is to D

E has to F a greater ratio than E to D [V. 13]

But that to which the same has a greater ratio is less. [V. 10]

F < D

D > F

Similarly,

If A = C
D = F

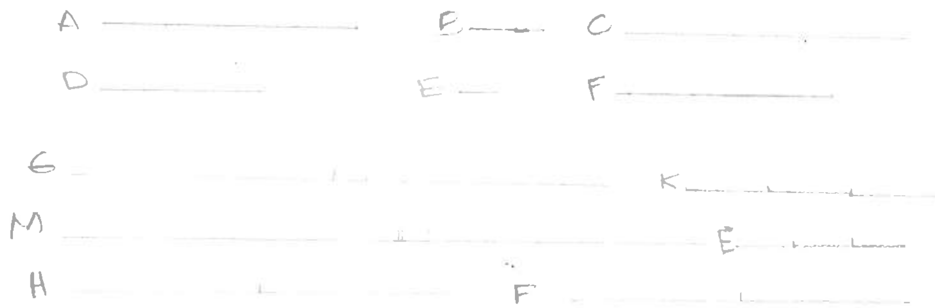
If less, less

Q.E.D.

Proposition 22

If there be any number of magnitudes whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, they will also be in the same ratio ex aequali.

given:



magnitudes A, B, C and D, E, F equal in multitude.
Taken two and two together are in the same ratio.

AS A is to B
D is to E

and as B is to C
E is to F

required:

They will be in the same ratio ex aequali
[V. def 17]

as A is to C
D is to F

G, H equimultiples of A, D
K, L equimultiples of B, E
M, N equimultiples of C, F

AS A is to B
D is to E

G, H equimultiples of A, D
K, L equimultiples of K, L

as G is to K
H is to L [V.4]

For the same reason,

as K is to M
L is to N

there are three magnitudes G, K, M and others H, L, N equal to them in multitude; which taken two and two together are in the same ratio,
↳ ex aequalis

If G is in excess of M
H is in excess of N
If equal, equal
if less, less [V.20]

G, H equimultiples of A, D
M, N equimultiples of C, F

As A is to C
D is to F [V. def. 5]

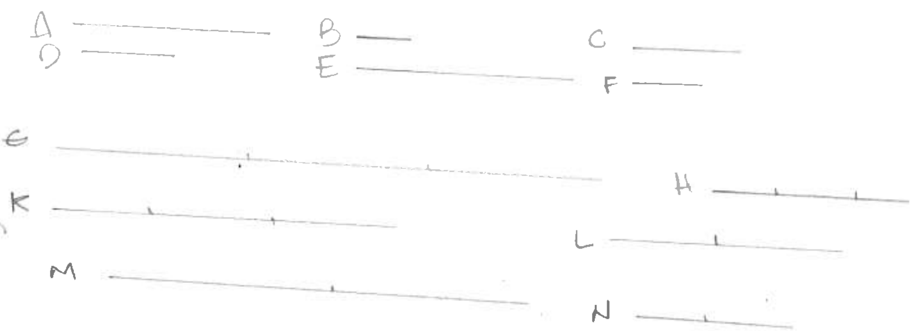
Q.E.D.

Proposition 23

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio ex aequali.

Given

magnitudes A, B, C and others equal to them in multitude, which taken two and two together, are in the same proportion, D, E, F and let the proportion be perturbed [V.def 18]



so, as A is to B
E is to F

and as B is to C
D is to F

required

as A is to C
D is to F

G, H, K equimultiples of A, B, D
L, M, N equimultiples of C, E, F

G, H equimultiples of A, B
Parts have the same ratio as the same multiples of them [V.15]
As A is to B
G is to H

For the same reason,
as E is to F
M is to N

As A is to B
E is to F
As G is to H
M is to N [V.11]

As B is to C
D is to E

alternately, B is to D
C is to E [V.16]

H, K equimultiples of B, D
Parts have the same ratio as their equimultiples
As B is to D
H is to K [V.15]

As B is to D
C is to E → As H is to K
C is to E [V.11]

L, M equimultiples of C, E
As C is to E
L is to M [V.15]

As C is to E
H is to K → As H is to K
L is to M [V.11]

alternately,
as H is to L
K is to M [V.16]

it was proved that
as G is to H
M is to N

Three magnitudes G, H, L, and others equal to them in multitude M, N, L, which taken two and two together are in the same ratio and the proportion is perturbed

→ ex aequali if G is in excess of L
K is in excess of N [V.21]

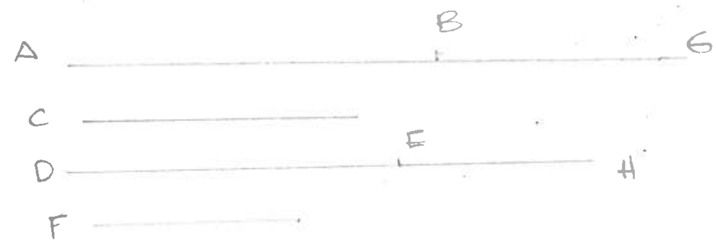
G, K equimultiples of A, D / L, N of C, F.
As A is to C, D is to F Q.E.D.

Proposition 24

If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.

given

first magnitude AB have to a second C, the same ratio as a third DE has to a fourth F; a fifth BG have to the second C the same ratio as a sixth EH has to a fourth F.



required

first and fifth added together, AG, will have to the second, C the same ratio that the third and sixth, DH have to the fourth F.

AS BG is to C
EH is to F

AS AB is to C
DE is to F

AS C is to BG
F is to EH

↳ ex aequali

AS AB is to BG
DE is to EH [V.22]

since the magnitudes are proportional separando, they will be proportional componendo [V.18]

AS AG is to GB
DH is to EH

also,

AS BG is to C
EH is to F

ex aequali

AS AG is to C
DH is to F [V.22]

Q.E.D

Proposition 25

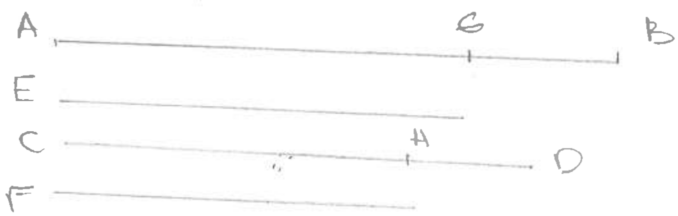
If four magnitudes be proportional, the greatest and the least are greater than the remaining two.

given:

magnitudes AB, CD, E, F be proportional.

AS AB is to CD
E is to F

AB The greatest
F The least



required

AB, F > CD, E

AG = E
CH = F

AS AB is to CD
E is to F

E = AG
F = CH

AS AB is to CD
AG is to CH

as The whole AB is to the whole CD, so is the part AG subtracted to the part CH subtracted.
The remainder GB will also be to the remainder HD as The whole AB is to the whole CD [V, 19]

AB > CD

GB > HD

AG = E
CH = F

AG, F = CH, E

GB, HD are unequal

GB > HD

AG, F added to GB
CH, E added to HD

AB, F > CD, E

Q.E.D