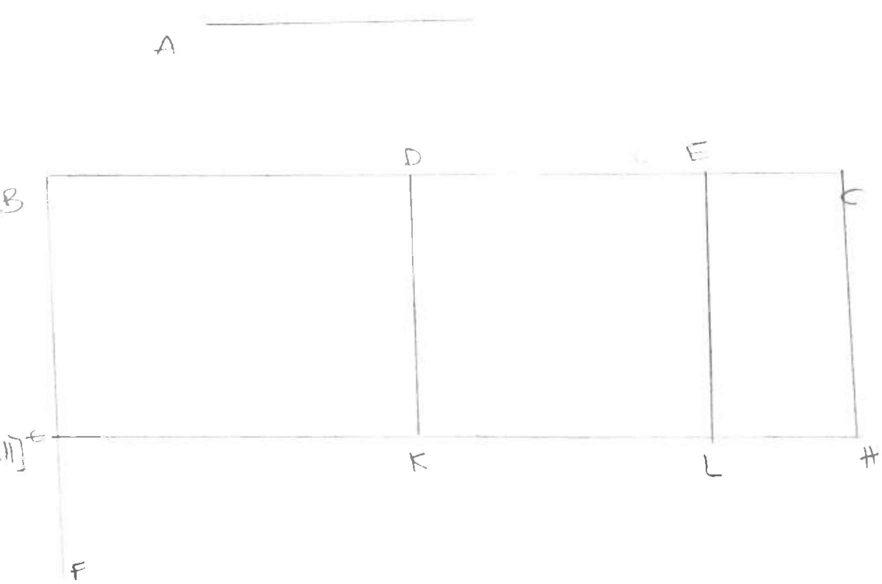
Proposition 1

If there be two straight lines, and one of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.

given:

$\overline{A}$, $\overline{BC}$

BC be cut at random at D, E



required:

The rectangle contained by A, BC is = Rectangle contained by A, BD, A, DE, A, EC.

BF at right angles to BC [I.11]

BG = A [I.3]

GH ∥ BC [I.31]

$\overline{DK}$, $\overline{EL}$, $\overline{CH}$ ∥ BG

□BH = BK, DL, EH

□BH = A, BC
contained by BG, BD
GB = A

□BK = A, BC
contained by GB, BD
BG = A

□DL = A, DE
contained by DK, DE [I.34]
DK = BG = A

□EH = A, DE
contained by EC, EL
EL = BG = A

□A, BC = □A, BD, □A, DE, □A, EC

Q.E.D

Proposition 2

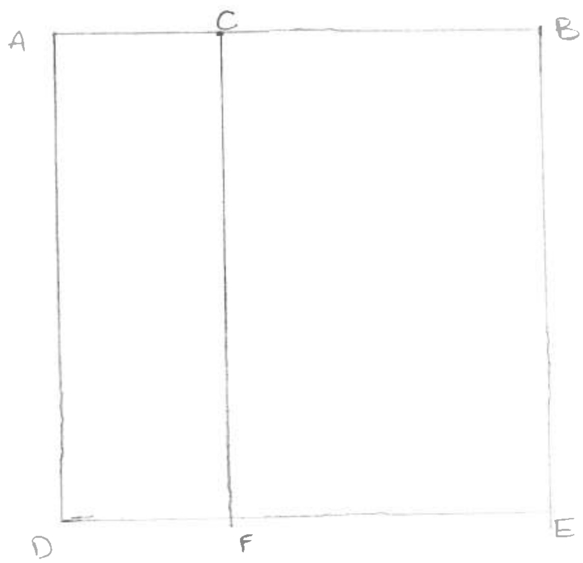
If a straight line be cut at random, the rectangles contained by the whole and both of the segments are equal to the square on the whole.

given:

$\overline{AB}$ cut at random at C.

required:

▯ contained by AB, BC & ▯ contained by BA, AC = ▯AB



▯ADEB [I.46]

CF ∥ AD, BE [I.31]

AE = AF, CE

AE is a square on AB

▯AF is contained by BA, AC
For it is contained by DA, AC
AD = BA

▯CE is contained by AB, BC
BE = AB

BA, AC + AB, BC = AE

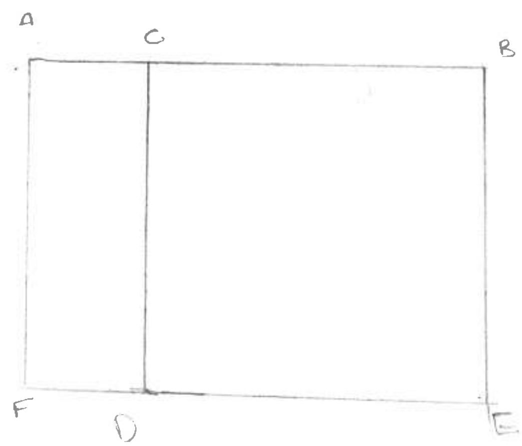
Q.E.D.

Proposition 3

If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square on the aforesaid segment.

given

$\overline{AB}$ be cut at random at C



Required

AB, BC =
AC, CB, □ on BC

□ CDBE on CB [I.46]
ED → F
AF ∥ CD, BE [I.31]

□AE = AD, CE

□ AE is contained by AB, BC
for it is contained by AB, BE
BE = BC.

□AD is contained by AC, CB
DC = BC

□B is a square on CB

□AB, BC = AC, CB + square BC.

QED

Proposition 4

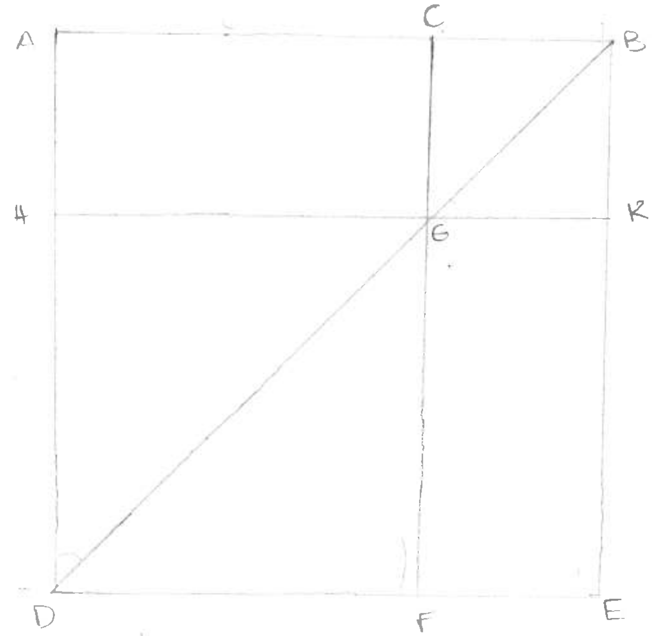
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments

given:

$\overline{AB}$ cut at C

required

Square AB = squares AC, CB and twice rectangle AC, CB



square ADEB [I.46]

$\overline{BD}$

CF ∥ BE, AD
HK ∥ AB, DE [I.31]

∡CGB = ∡ADB
CF∥AD, BD has fallen on them [I.29]

∡ADB = ∡ABD
BA = AD [I.5]

∡GCB = ∡GBC
BC = CG [I.6]

CB = GK
CG = KB [I.34]
GK = KB
CGKB is equilateral

It is also right-angled
CG∥BK
∡KBC, ∡GCB = 2∟ [I.29]
∡KBC is right
∡BCG is right (opposite)
∡CGK, ∡GKB are right [I.34]

CGKB is right angled an equilateral
→It is a square on CB

For the same HF is a square on HG.
HG = AC [I.34]

HF, CK are squares on AC, CB

AG = GE
□AG is on AC, CB
GC = CB
□GE is on AC, CB

AG, GE = 2 □ AC, CB

HF, CK, AG, GE = squares on AC, CB and twice the rectangles AC, CB

HF, CK, AG, GE = ADEB (square AB)

→square on AB = squares on AC, CB & 2 □ AC, CB

Q.E.D

Proposition 5

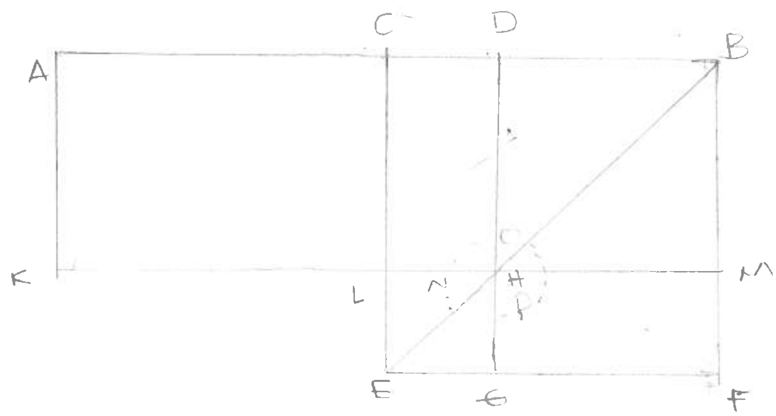
If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half.

given:

$\overline{AB}$ cut into equal segments at C and unequal segments at D

required:

▭ AD, DB & square CD = square on CB



square CEFB [I.46]

$\overline{BE}$

DG ∥ CE, BF
KM ∥ AB, EF [I.31]
AK ∥ CL, BM

complement CH = HF [I.43]

→ add DM

CH, DM = HF, DM

CM = DF

CM = AL

AC = CB [I.34]

AL = DF

→ add CH

AH = gnomon NOP

AH = AD, DB

DB = DH

NOP = AD, DB

→ add LG (LG = sq CD)

NOP, LG = AD, DB, square on CD

NOP, LG = CEFB (contained on CB

AD, DB, square on CD = square on CB

Q.E.D

Proposition 6

If a straight line be bisected and a straight line be added to it in a straight line, The rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.

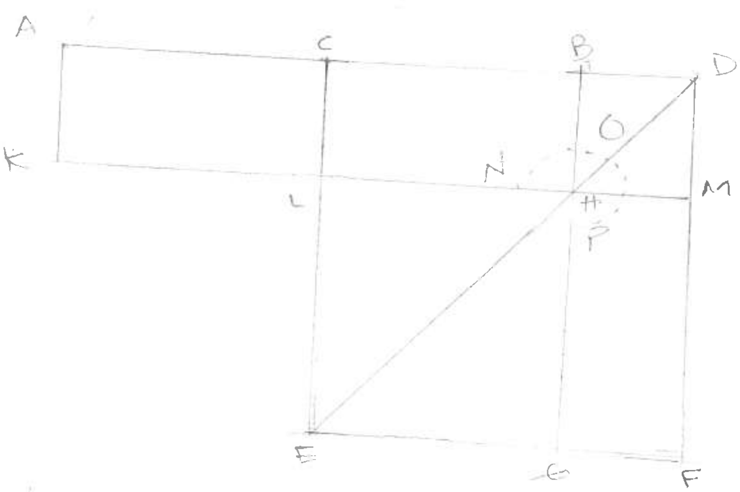
given:

$\overline{AB}$ bisected at C

$\overline{BD} \to \overline{AB}$

required:

rectangle contained by AD, DB, and the square on CB = square on CD



square CEFD on CD [I.46]

$\overline{DE}$

BG ∥ EC, DF

KM ∥ AB, EF [I.31]

AK ∥ CL, DM

AC = CB

AL = CH [I.36]

CH = HF

AL = HF

→ add CM

AL, CM = HF, CM

AM = NOP gnomon

AM = rectangle AD, DB

DM = DB

NOP = AD, DB

→ add LG (square on BC)

NOP, LG = AD, DB, square on BC

NOP, LG = CEFD (square on CD)

AD, DB, square on BC = square on CD

QED

Proposition 7

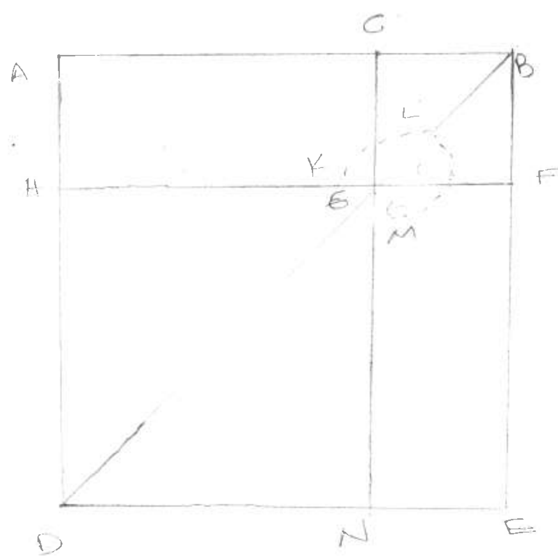
If a straight line be cut at random, the square on the whole and that on one of the segments both together are equal to twice the rectangle contained by the whole and the said segment and the square on the remaining segment.

given:

$\overline{AB}$ cut at random at point C.

required:

squares on AB, BC = twice the rectangle contained by AB, BC and square on AB



square ADEB on AB [I.46]
draw figure

AG = GE [I.43]
→add CF
AF = CE
AF, CE = double AF

AF, CE are the gnomon KLM & square on CF
KLM & square CF = double AF

twice AB, BC▯ = double AF
BF = BC
KLM & square on CF = twice AB, BC ▯
→add DG (square on AC)
KLM, square BG, square DG = twice AB, BG and square on AC

KLM and squares BG, DG = ADEB and CF (both squares on AB, BC)

squares AB, BC = twice AB, BC ▯ and square AC

QED

Proposition 8

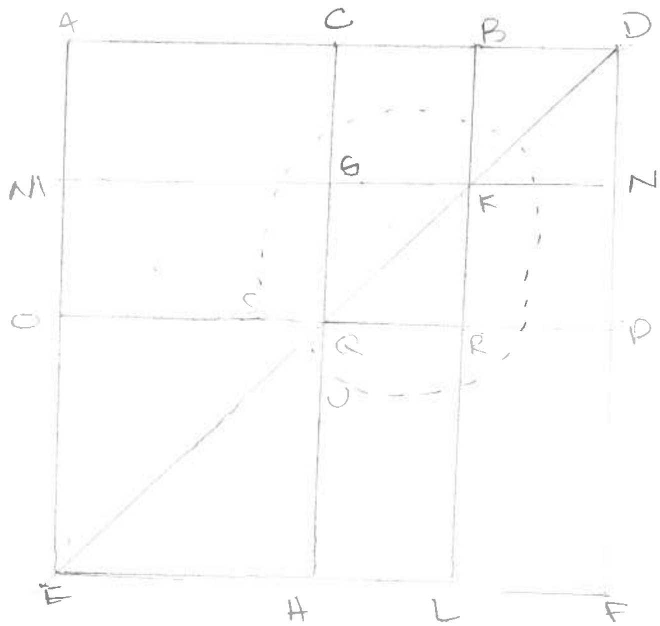
If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and the aforesaid segment as on one straight line.

given:

$\overline{AB}$ cut at random at C

required

4 times the rectangle contained by AB, BC
& square on AC
=
square on AB, BC as on one straight line.



BD → AB,
BD = CB
square AEFD on AD
draw figure

CB = BD
CB = GK
BD = KN
GK = KN
same reason
∴ QR = RP
BC = BD, GK = KN
CK = KD, GR = RN [I.36]

CK = RN
Both complements of ▱ CP [I.43]
KD = GR
KD = CK = GR = RN
↳ quadruples of CK

CB = BD
BD = BK = CG
CB = GK = GQ
CG = GQ.

CG = GQ,
QR = RP
AG = MQ [I.36]
QL = RF

MQ = QL
complements of ▱ ML [I.43]
AG = RF
MQ = QL = AG = RF
↳ quadruples of AG
but CK, KD, GR, RN are also quadruples of CK
These 8 areas contain gnomon STU and
quadruple AK

AK is rectangle AB, BD
BK = BD
4 times AB, BD = quadruple of AK
4 times AB, BD = gnomon STU
→ add OH (square on AC)
4 times AB, BD & square AC = STU & square AC
STU & square AC = AEFD (on AD)
4 times AB, BD & square AC = AEFD
BD = BC
4 times AB, BD, square AC = square on AD (AB, BC)

Q.E.D.

Proposition 9

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.

Given:

B cut into
equal
gments at C
nd unequal at
D

equired:

quares on AD, DB
re double the
quares on AC, CD

E = AC/CB
E at ∟ to AB
EA, EB

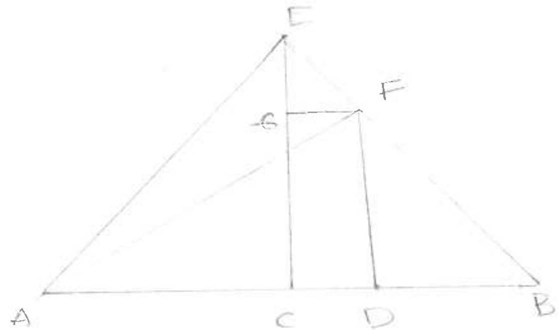
F ∥ EC
G ∥ AB
F

C = CE
EAC = ∡AEC
= ∟
EAC, ∡AEC = ∟ [I.32]
CEA = half ∟
CAE = half ∟
For the same reason
CEB = half ∟
EBC = half ∟

∡CEA, ∡CEB = ∡AEB = ∟

GEF = half ∟
EGF = ∟ = ∡ECB [I.29]
EFG = half ∟ [I.32]
GEF = ∡EFG
GE = GF [I.6]

B = half ∟
ECB = ∡FDB = ∟ [I.29]
BFD = half ∟ [I.32]
= ∡BFD



AC = CE
square on AC = square on CE
squares on AC, CE = 2 squares on AC
square EA = squares AC, CE
For ∡ ACE = ∟ [I.47]
square on EA = double square on AC

EG = GF
square on EG = square on GF
squares EG, GF = double square GF
square EF = squares EG, GF
square EF = double square GF

GF = CD [I.34]
EF = double square on CD
EA = double square on AC
squares on EA, EF = double squares on AC, CD

square on AF = squares on AE, EF
For ∡ AEF = ∟ [I.47]

square on AF = double squares AC, CD

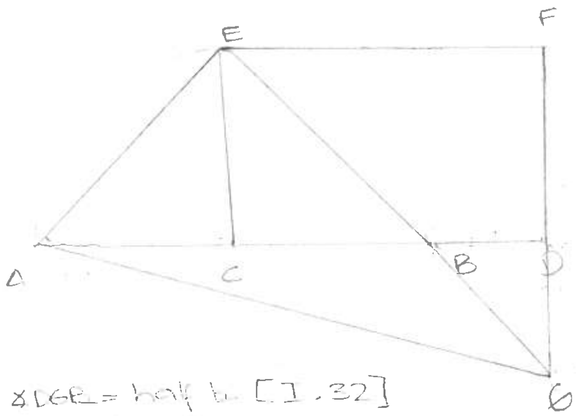
squares on AD, DF = double squares on AC, CD

DF = DB
squares AD, DB = double squares AC, CD.

Q.E.D

Proposition 10

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of half and the added straight line as on one straight line.



given

AB bisected at C.
Add BD in a straight line

required

squares on AD, DB are double of the squares on AC, CD.

CE at right angle with AB [I. 11]

CE = AC, CB [I. 3]

EA, EB

EF ∥ AD
FD ∥ CE [I. 31]

∡CEF, ∡EFD = 2∟ [I. 29]
They fall on the same parallels

∡FEB, ∡EFD < 2∟

But straight lines produced from angles less than 2∟ meet [Post 5]

EB & FD, if produced, meet at G
AG

AC = CE
∡EAC = ∡AEC [I. 5]
∡C = ∟
∡EAC = half ∟
∡AEC = half ∟

For the same
∡CEB = half ∟
∡CBE = half ∟
∡AEB = ∟

∡BGD = half ∟
For ∡EBC is half ∟

∡BDG = ∟ [I. 29]
For ∡BDG = ∡DCE
(They are alternate)

∡DGB = half ∟ [I. 32]
∡DGB = ∡GBD
BD = GD [I. 6]

∡EGF = half ∟
∡F = ∟
For ∡F = ∡C [I. 34]
For they are opposite
∡FEG = half ∟ [I. 32]
∡EGF = ∡FEG
GF = EF [I. 6]

square on EC = square on CA
squares on EC, CA = 2 square on CA
square on EA = squares on EC, CA [I. 47]
square on EA = 2 square on CA (C.N. 1)

FG = EF
square on FG = square on EF
squares FG, EF = 2 square EF
square EG = squares FG, GF [I. 47]
square EG = 2 square EF

EF = CD [I. 34]
square EG = 2 square CD

square EA = 2 square AC

squares EA, square EG = 2 square AC, 2 square CD

square AG = squares AE, EG [I 47]

square AG = 2 square AC, 2 square CD

squares AD, DG = square AG [I 47]

squares AD, DG = 2 square AC, 2 square CD

DG = DB

squares AD, DB = 2 square AC, 2 square CD

Proposition 11

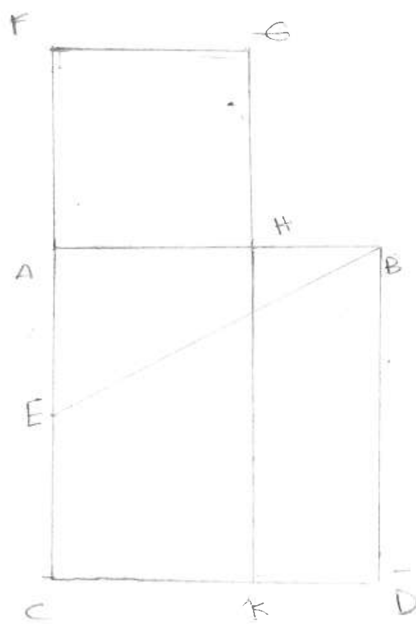
to cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

given:

AB

required

cut AB so that
the rectangle contained
by the whole & one of
the segments =
square on remaining
segment



square ABCD [I.46]

bisect AC at E

E

AF → CA

EF = BE

square FH

GH → K

AB has been cut at H
to make the rectangle
contained by AB, BH =
to the square on AH

* (says)
AB cut at H to make
rectange AB, BH =
square AH

rectangle CF, FA &
square AE
= square EF [II.6]

EF = EB
rectangle CF, FA & square AE
= square EB

squares BA, AE = square EB
for ∡A is right [I.47]

rectangle CF, FA, square AE = square BA, AE
→ subtract square AE
rectangle CF, FA = square BA, AE

rectangle CF, FA = FK
for: AF = FG
square AD is on AB
FK = AD

→ subtract AK
FH = HD

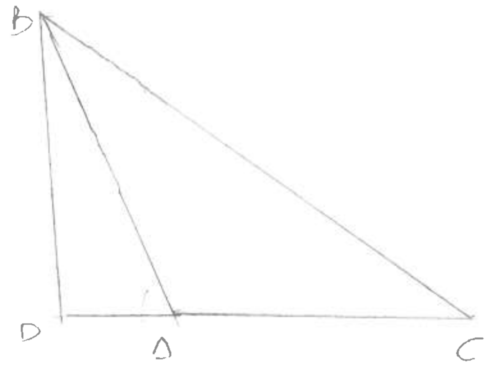
HD = rectangle AB, BH
for AB = BD
FH is a square on HA
rectangle contained on AB, BH = square on HA

Therefore, AB has been cut at H to
make a rectangle contained by AB, BH
= square on AH

Proposition 12

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.

given:



△ABC obtuse-angled
∡BAC obtuse
$\overline{BD}$ drawn from point B perpendicular to CA produced

required:

Square on BC > squares on BA, AC by twice the rectangle contained by CA, AD

CD has been cut at random at A

Square DC = squares CA, AD and twice the rectangle CA, AD [II.4]

→ add square DB

Squares DC, DB = squares CA, AD, DB and twice the rectangle CA, AD

Square on CB = squares on CD, DB [I.47]
For ∡D is ⊾

Square AB = squares on AD, DB [I.47]

Square on CB = squares on CA, AB and twice the rectangle CA, AD

Square CB > squares CA, AB by twice the rectangle CA, AD

Q.E.D

Proposition 13

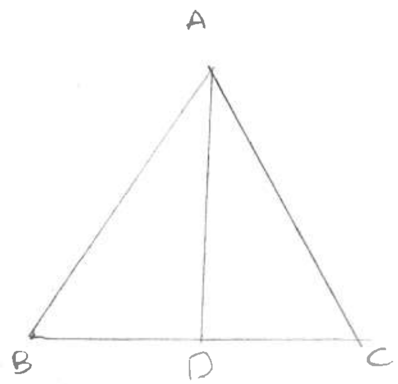
In acute-angle triangles the square on the side subtending the acute angle is less than the squares on the sides containing the acute angle by twice the rectangle contained by one of the sides about the acute angle, namely that on which the perpendicular falls, and the straight line cut off within the perpendicular towards the acute-angle

given:

△ABC

∡B acute

$\overline{AD}$ be drawn from A perpendicular to BC



required:

square AC < squares on CB, BA by twice the rectangle contained by CB, BD

For CB was cut at random at D

squares CB, BD = 2 rectangle CB, BD and square on DC [II.7]

+ add square DA

squares CB, BD, DA = squares DC, DA & 2 rectangle CB, BD

square AB = squares on BD, DA [I.47]
for ∡D is rt.

square AC = squares AD, DC

squares CB, BA = square AC & 2 rectangle CB, BD

square AC < squares CB, BA
by 2 rectangle CB, BD

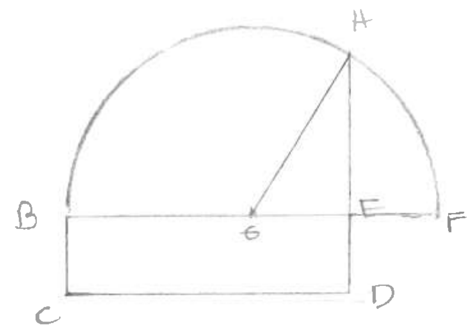
Q.E.D.

Proposition 14

to construct a square equal to a given rectilineal figure.

given.
rectilineal figure A

required
□ = rectilineal figure A



▱BD = rectilineal figure A [I.45]

if BE=ED There would be a square = rectilineal figure A

But if not
BE > ED
BE produced to F
EF = ED
Bisect BF at G

⊙ center G, distance GB

DE → H
GH

since BF has been cut into equal segments at G, and unequal at E.
rectangle BE,EF, square EG = square GF [II.5]

GF = GH
rectangle BE,EF, square GE = square GH

Squares HE,EG = square GH
rectangle BE,EF, square GE = squares HE,EG
→ subtract square EG
rectangle BE,EF = square HE

rectangle BE,EF = BD
for EF = ED

▱BD = square HE

BD = rectilineal figure A
rectilineal figure A = square HE

Q.E.F.