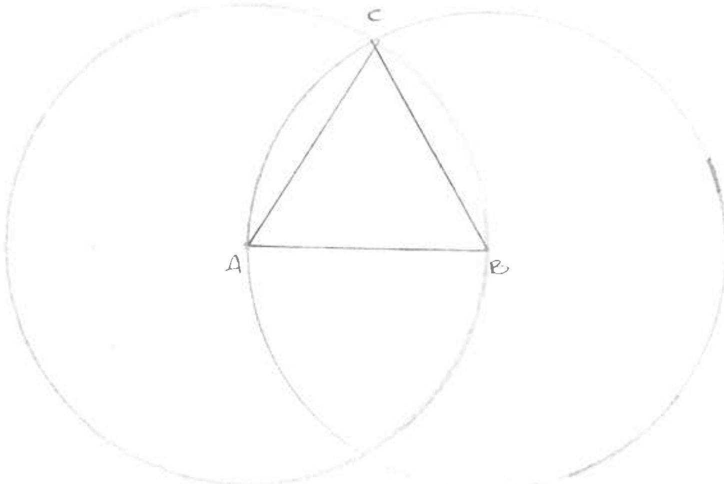
Proposition 1

On a given finite straight line to construct an equilateral triangle

Given: $\overline{AB}$
Required: $\triangle$



$\overline{AB}$

$\odot BCD$ [Post 3]

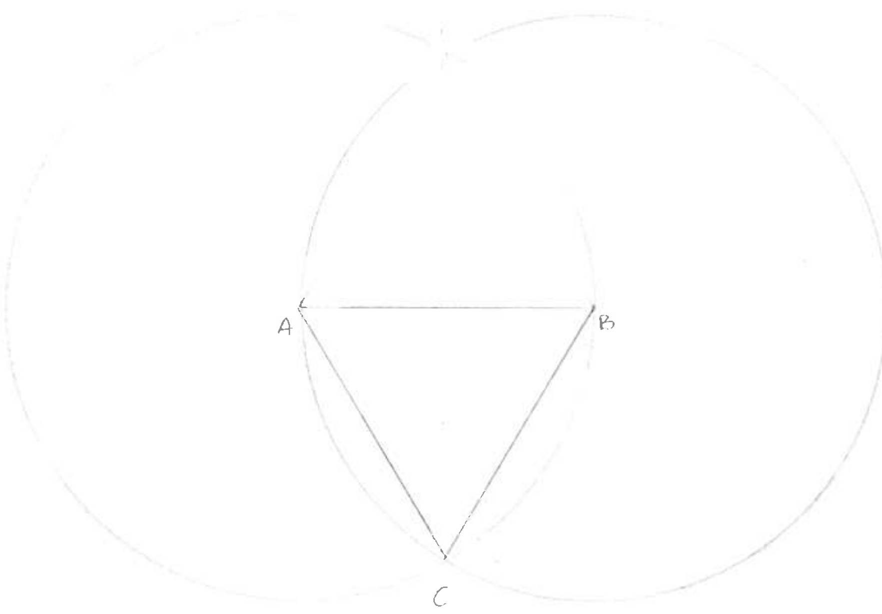
$\odot ACE$ [Post 3]

$\overline{CA}, \overline{AB}$ [Post 1]

$\overline{AC} = \overline{AB}$ [Def 15]

$\overline{BC} = \overline{AB}$ [def. 15]

$\overline{AC} = \overline{BC}$ [C.N. 1]

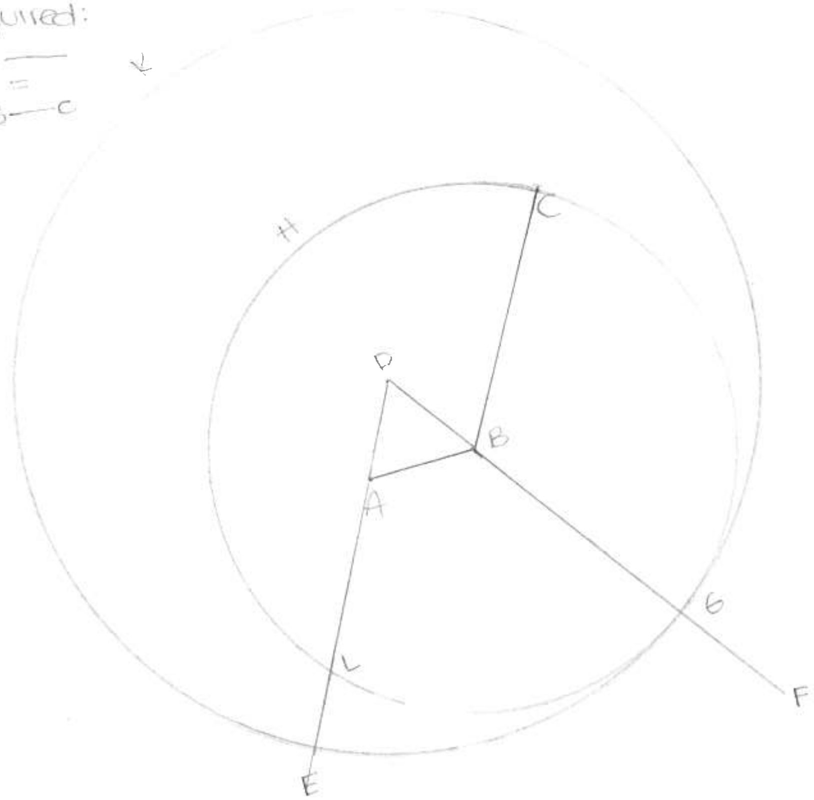


Proposition 2

To place at a given point [as an extremity] a straight line equal to a given straight line.

Given: A
$\overline{BC}$

Required:
A——
=
B——C



.A $\overline{BC}$

$\overline{AB}$ [Post 1]

$\triangle$ [I, 1]

$\overline{AE}, \overline{BF}$ [Post.

⊙CGH [Post 3]

⊙GKL [Post 3]

$\overline{BC} = \overline{BG}$ [def. 15]

$\overline{DL} = \overline{DG}$

$\overline{DA} = \overline{DB}$

$\overline{AL} = \overline{BG}$ [C N 3]

$\overline{AL} = \overline{BG}$ [C N 1]

Proposition 3

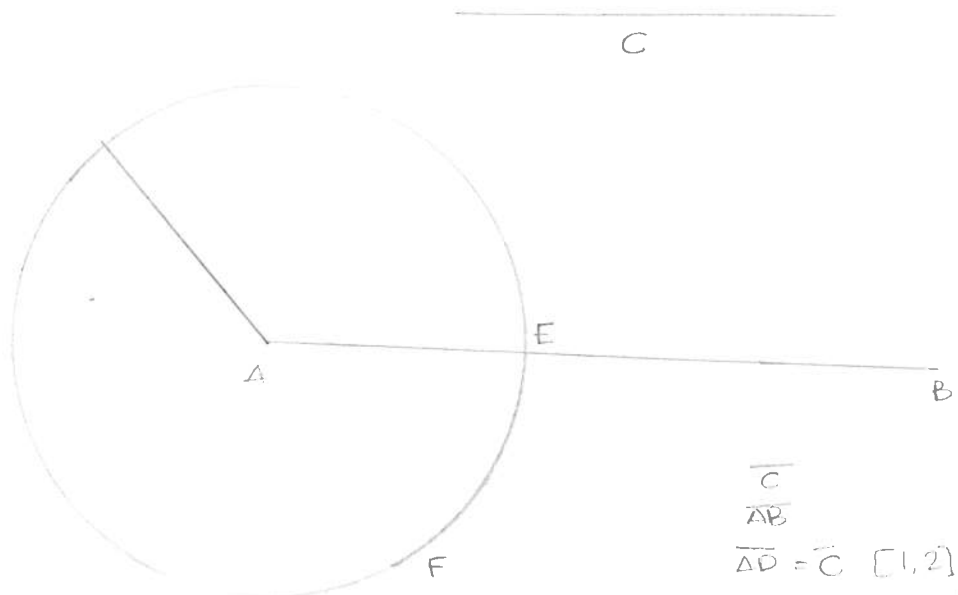
Given two unequal straight lines, to cut off from the greater a straight line equal to the less.

Given:

$\overline{AB}$

$\overline{C}$

Required:



$\overline{C}$

$\overline{AB}$

$\overline{AD} = \overline{C}$ [1, 2]

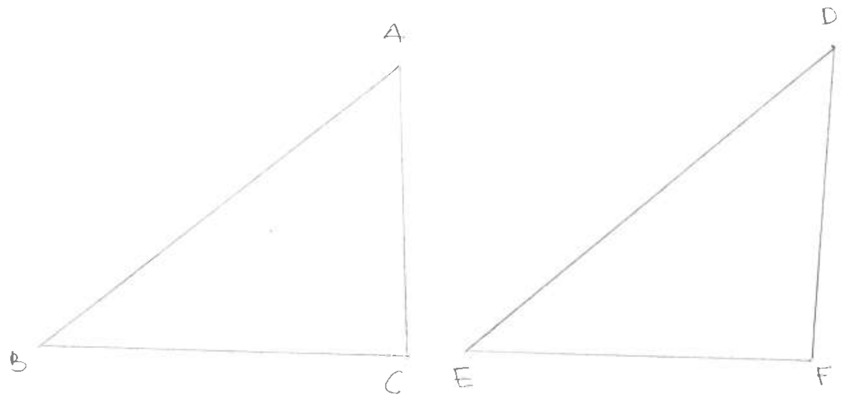
$\odot DEF$ [Post 3]

$\overline{AE} = \overline{AD}$ [def. 15]

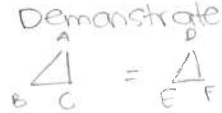
$\overline{AE} = \overline{C}$ [C.N. 1]

Proposition 4

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.



Demonstrate:



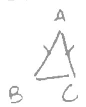
BC = EF
A = D
AB = DE
B = E
AB = DE
AC = DF
∡BAC = ∡EDF
C = F

B = E
BC = EF [C.N.4]
∡ ABC = △DEF [C.N.4]
∡ = ∡
∡ABC = ∡DEF [C.N.4]
∡ACB = ∡DFE

Proposition 5

In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.

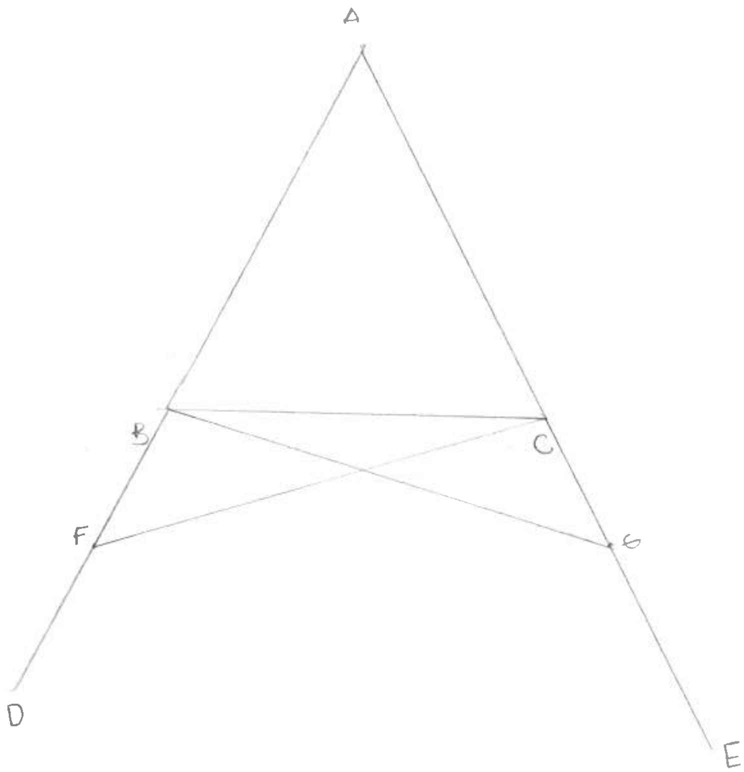
Given:



Demonstrate:

$\measuredangle ABC = \measuredangle ACB$

$\measuredangle CBD = \measuredangle BCE$



$\triangle ABC$

$AB = AC$

$\overline{BD}, \overline{CE}$ [Post. 2]

$\cdot F$

$AG = AF$ [1.3]

$\overline{FC}, \overline{GB}$ [Post. 1]

$\overline{AB} = \overline{AC}$

$\overline{AF} = \overline{AG}$

$\triangle GAB = \triangle AFC$

$\measuredangle ACF = \measuredangle ABG$

$\measuredangle AFC = \measuredangle AGB$ [1.4]

$\overline{BF} = \overline{CG}$ [C.N. 3]

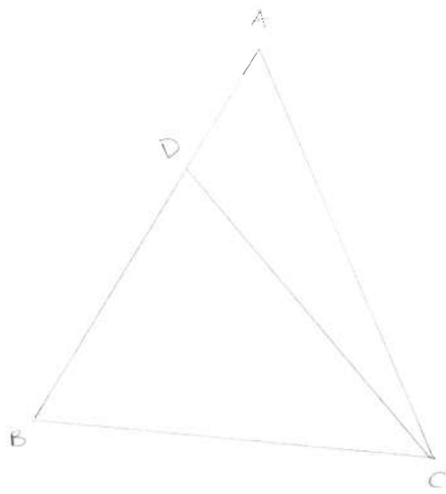
$\triangle BFC = \triangle BCG$ [1.4]

$\overline{BC} = \overline{CB}$

$\measuredangle FBC = \measuredangle CGB$

Proposition 6

If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another



$\overline{DB} = AC$

$\overline{BC} = \overline{CB}$

$\overline{DB} = \overline{AC}$

$\measuredangle DBC = \measuredangle ACB$

$\overline{DC} = \overline{AB}$

$\triangle DBC = \triangle ACB$

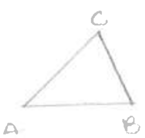
is absurd

so $AB = AC$

Proposition 7

Given two straight lines constructed on a straight line [from its extremities] and meeting in a point, there cannot be constructed on the same straight line [from its extremities], and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it.

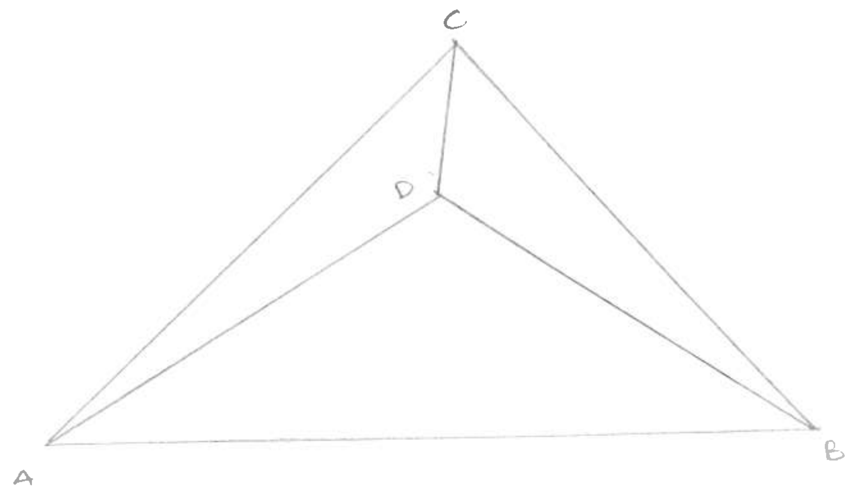
Given:



Demonstrate:

$AC \neq AD$

$BC \neq BD$



$\overline{CA} = \overline{DA}$

$\overline{CB} = \overline{DB}$

$\overline{CD}$

$\overline{AC} = \overline{AD}$

$\measuredangle ACD = \measuredangle ADC$

$\measuredangle \overset{CDB}{ADC} > \measuredangle DBC$

$CB = DB$

$\measuredangle CDB = \measuredangle DCB$

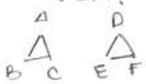
Impossible!

Q.E.D.

Proposition 8

If two triangles have two sides equal to two sides respectively, and have also the base equal to the base, they will also have the angles equal which are contained by the equal straight lines.

Given:



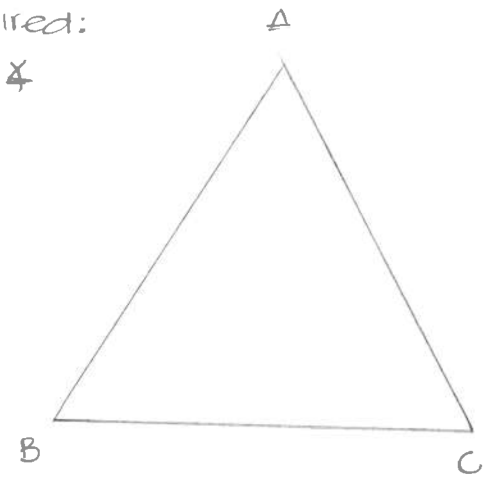
$\overline{AB} = \overline{DE}$

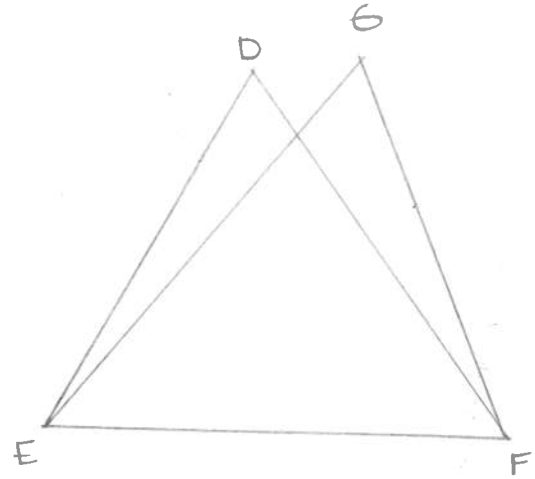
$\overline{AC} = \overline{DF}$

$\overline{BC} = \overline{EF}$

required:

$\measuredangle = \measuredangle$





$\overline{AB} \neq \overline{DE}$

$\overline{AC} \neq \overline{DF}$

$\overline{AB} = \overline{EG}$

$\overline{AC} = \overline{GF}$

cannot be constructed

$\measuredangle BAC = \measuredangle EDF$

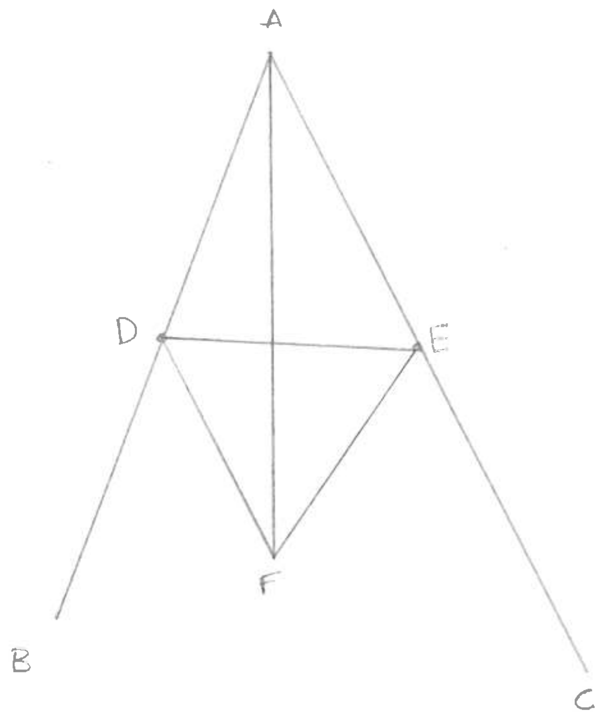
QEF

Proposition 9

To bisect a given rectilineal angle.

Given:
$\measuredangle BAC$

required:
Bisect $\measuredangle BAC$.



$\cdot D$

$\overline{AE}$

$\overline{AE} = \overline{AD}$

$\overline{DE}$

$\triangle DEF$

$\overline{AF}$

$\overline{DA} = \overline{EA}$

$\overline{AF} = AF$

$\overline{DF} = \overline{EF}$

$\measuredangle DAF = \measuredangle EAF$

QED

Proposition 10

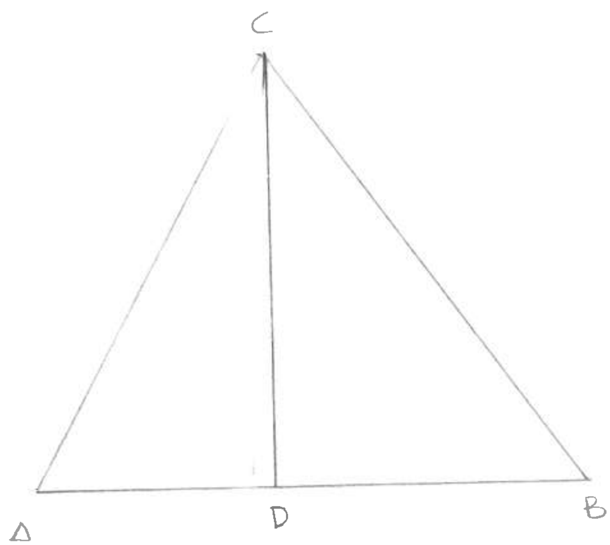
To bisect a given finite straight line.

Given:

$\overline{AB}$

required:

bisect $\overline{AB}$



$\triangle ABC$ [1,1]

$\overline{CD}$ [1,9]

$\overline{AC} = \overline{CB}$

CD common

$\overline{AC} = \overline{BC}, \overline{CD} = \overline{CD}$

$\measuredangle ACD = \measuredangle BCD$

$\overline{AD} = \overline{BD}$

QEF.

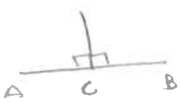
Proposition 11

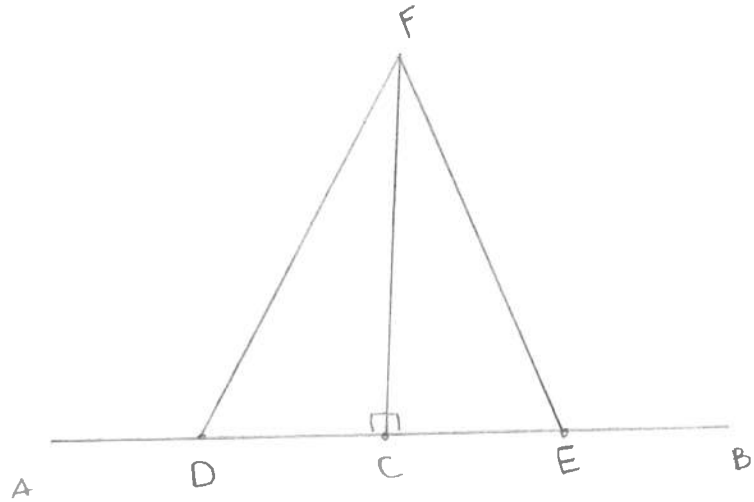
To draw a straight line at right angles to a given straight line from a given point on it.

given:

$\overline{AB}$ $\cdot C$

required:





$\cdot D$

$\overline{CE} = \overline{CF}$ [1,3]

$\triangle FDE$ [1,1]

$\overline{FC}$

$\overline{DC} = \overline{CE}$

$\overline{CF}$ common

$\overline{DC} = \overline{CE}$, $\overline{CF} = \overline{CF}$

$\overline{DF} = \overline{EF}$

$\measuredangle DCF = \measuredangle ECF$ [1,8]

$\measuredangle DCF$ & $\measuredangle ECF$ $= \llcorner$ [def.10]

Q.E.F

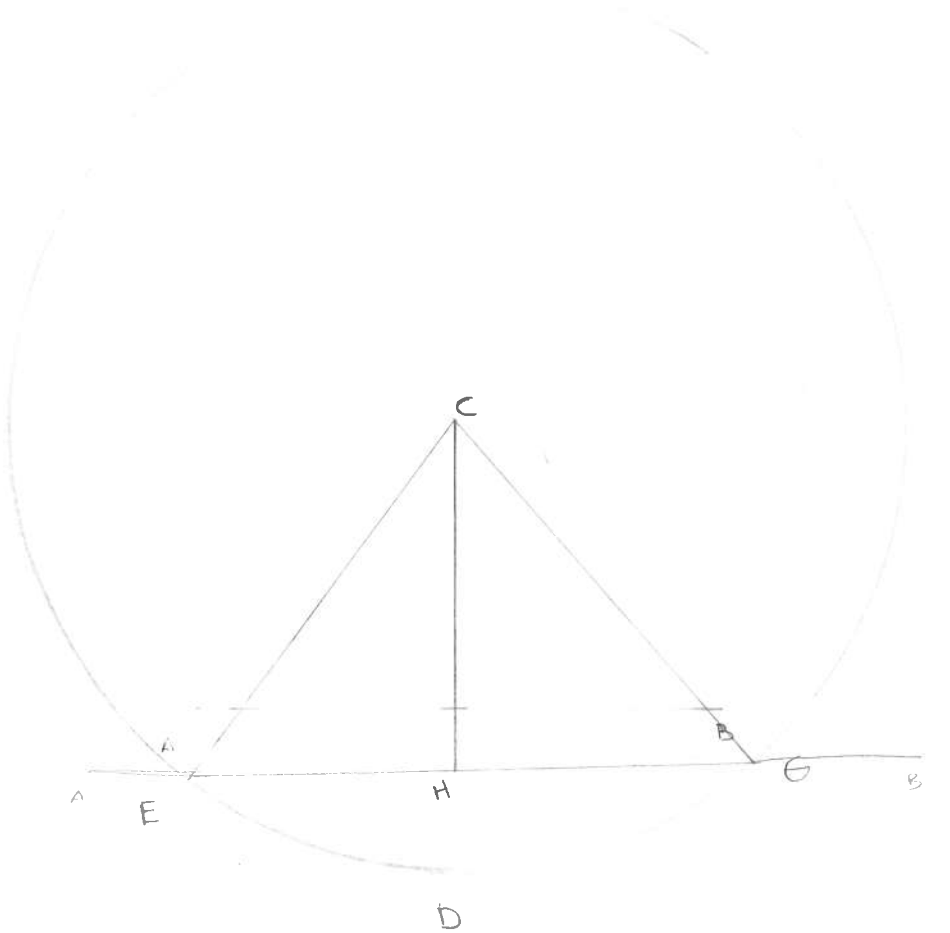
Proposition 12

to a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.

given:

$\overline{AB}$ ·C

required:



·D

○EFG [Post 3]

$\overline{EG}$ bisected H [1,10]

$\overline{CG}$, $\overline{CH}$, $\overline{CE}$ [Post 1]

$\overline{GH} = \overline{HE}$ & $\overline{HC}$ common

$\overline{HG} = \overline{HE}$ & $\overline{HC} = \overline{CH}$

$\overline{CG} = \overline{CE}$

$\measuredangle CHG = \measuredangle EHG$ [1,8]

$\measuredangle = \llcorner$ [def. 10]

QEF

Proposition 13

If a straight line set up on a straight line make angles, it will make either two right angles or angles equal to two right angles.

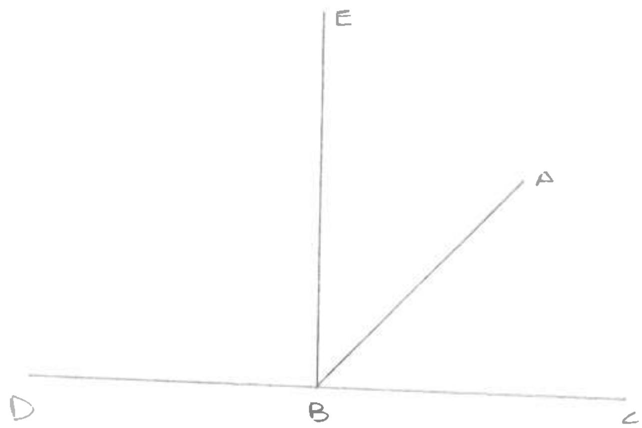
given

$\measuredangle CBA$, $\measuredangle ABD$

required

$2\llcorner =$

$\measuredangle CBA$, $\measuredangle ABD$



$\measuredangle CBA = \measuredangle ABD \rightarrow 2\llcorner$ [def 10]

$\overline{BE}$

$\measuredangle CBE, \measuredangle EBD = 2\llcorner$ [I.11]

$\measuredangle CBE = \measuredangle CBA, \measuredangle ABE$

$\rightarrow$ add EBD

$\measuredangle CBE, \measuredangle EBD = \measuredangle CBA, \measuredangle ABE, \measuredangle EBD$ [C.N.2]

$\measuredangle DBA = \measuredangle DBE, \measuredangle EBA$

$\rightarrow$ add ABC

$\measuredangle DBA, \measuredangle ABC = \measuredangle CBA, \measuredangle ABE, \measuredangle EBD$ [C.N.2]

$\measuredangle CBE = \measuredangle EBD$ [C.N.1]

$\rightarrow \measuredangle CBE, \measuredangle EBD = \measuredangle DBA, \measuredangle ABC$

$\measuredangle DBA, \measuredangle ABC = 2\llcorner$

Q.E.D

Proposition 14

If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal to two right angles, the two straight lines will be in a straight line with one another.

given:

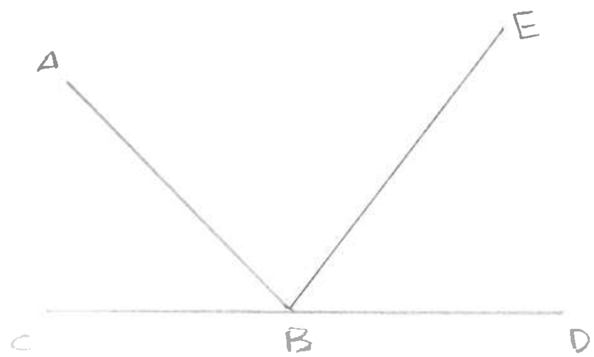
$\overline{AB}$ $\cdot B$

$\overline{BC}, \overline{BD}$ make 2 adjacent angles ✓

$\measuredangle ABC, \measuredangle ABD = 2 \llcorner$

required:

$\overline{BD}$ is in a straight line with $\overline{CB}$



suppose:

$\overline{BE}$ is in a straight line with $\overline{CB}$

$\measuredangle ABC, \measuredangle ABE = 2 \llcorner$ [I.13]

But

$\measuredangle ABC, \measuredangle ABD = 2 \llcorner$

$\measuredangle ABC, \measuredangle ABE = \measuredangle ABC, \measuredangle ABD$ [Post 4] & [C N 1]

→ Subtract $\measuredangle ABC$

$\measuredangle ABE = \measuredangle ABD$ [C N 3]

impossible by C N 5

$\overline{BE}$ is not in straight line with CB

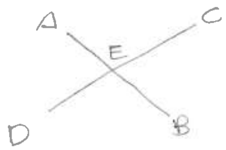
$\overline{CB}$ is in straight line with $\overline{BD}$

QED

Proposition 15

If two straight lines cut one another, they make the vertical angles equal to one another.

Given:

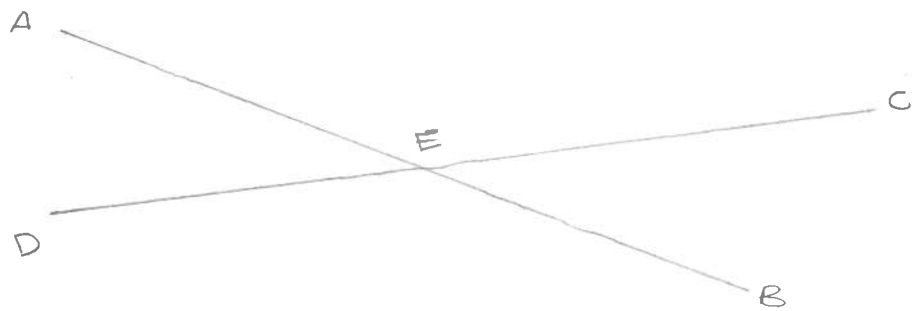


let line $\overline{AC}$ & $\overline{AD}$ cut each other at E.

required:

$\measuredangle AEC = \measuredangle DEB$

$\measuredangle CEB = \measuredangle AED$



$\measuredangle CEA, \measuredangle AED = 2\llcorner$ [I.13]

$\measuredangle AED, \measuredangle DEB = 2\llcorner$ [I.13]

$\measuredangle CEA, \measuredangle AED = \measuredangle AED, \measuredangle DEB$ [Post. 4] [C.N. 1]

→ subtract $\measuredangle AED$

$\measuredangle CEA = \measuredangle DEB$ [C.N. 3]

Similarly, $\measuredangle CEB = \measuredangle DEA$

Q.E.D.

Proposition 16

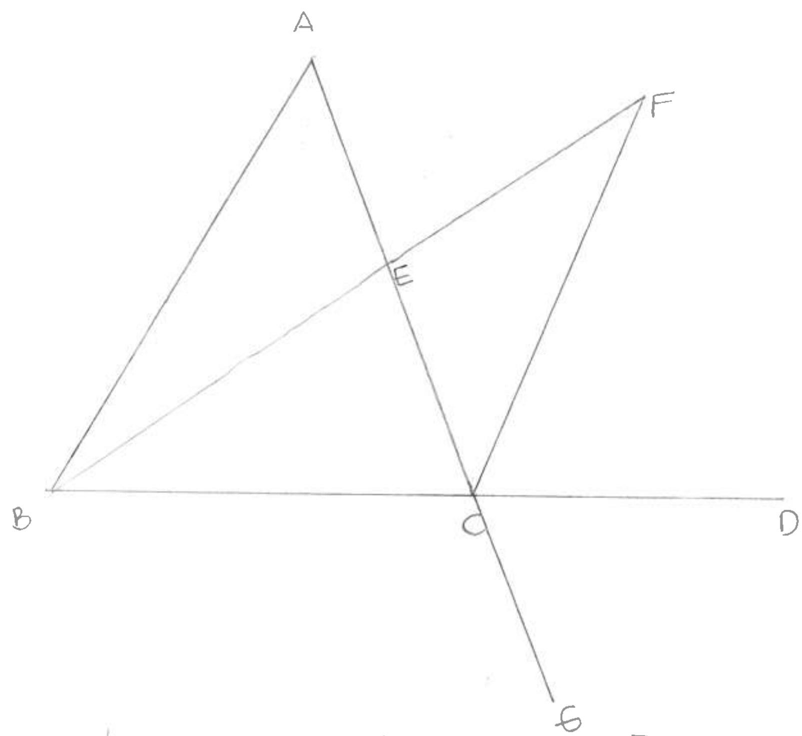
In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.

Given:

$\triangle ABC$
$\overline{CD}$

Required:

$\measuredangle ACD > \measuredangle CBA, \measuredangle BAC$



Bisect $\overline{AC}$ at E [I.10]
$\overline{BE}, \overline{EF}$ [Post. 1]
$\overline{EF} = \overline{BE}$ [I.3]
$\overline{FC}$ [Post. 1]
extend $\overline{AC}$ to G [Post. 2]

$\overline{AE} = \overline{EC}$
$\overline{BE} = \overline{EF}$
$\overline{AE}, \overline{EB} = \overline{EC}, \overline{EF}$
$\measuredangle AEB = \measuredangle FEC$ [I.15]
$\overline{AB} = \overline{CF}$
$\triangle ABE = \triangle CFE$ [I.4]
$\measuredangle BAE = \measuredangle ECF$

$\measuredangle ECD > \measuredangle ECF$ [C.N.5]
$\measuredangle ACD > \measuredangle BAE$

Q.E.D

Proposition 17

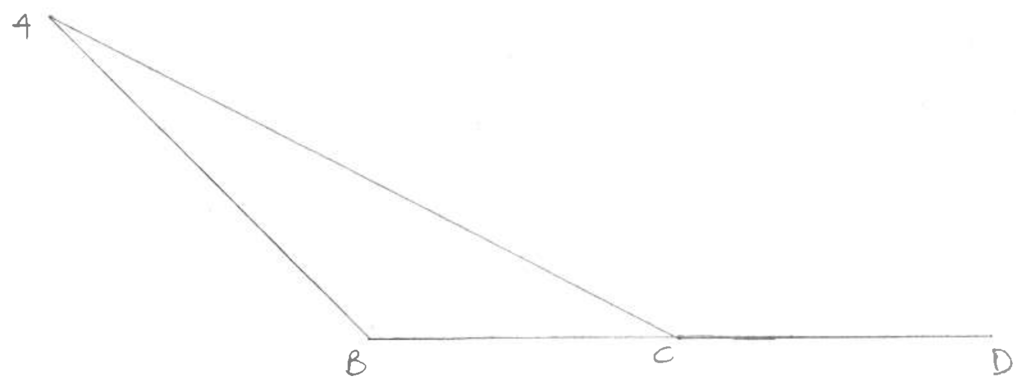
In any triangle two angles taken together in any manner are less than two right angles.

Given.

$\triangle ABC$

required

2 angles of $\triangle ABC$ are less than 2∟



$\overline{CD}$ [post 2]

$\measuredangle ACD > \measuredangle ABC$ [1.16]

add $\measuredangle ACB$

$\measuredangle ACD, \measuredangle ACB > \measuredangle ABC, \measuredangle ACB$

$\measuredangle ACD, \measuredangle ACB = 2$∟ [1.13]

$\measuredangle ABC, \measuredangle ACB < 2$∟

QED

similarly $\measuredangle BAC, \measuredangle ACB < 2$∟

$\measuredangle CAB, \measuredangle ABC < 2$∟

Proposition 18

In any triangle the greater side subtends the greater angle.

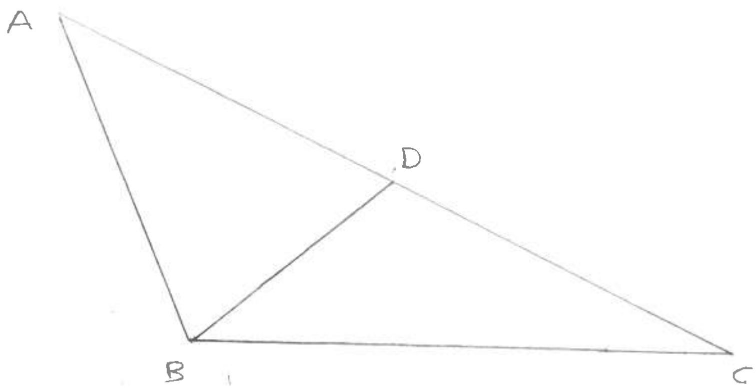
Given

ΔABC

$AC > AB$

required

$\measuredangle ABC > \measuredangle BCA$



$AD = AB$ [1.3]

BD [post. 1]

$\measuredangle ABD$ is an exterior angle of $\measuredangle BCD$

$\measuredangle ADB > \Delta DCB$ [1.16]

$\measuredangle ADB = \measuredangle ABD$

$\measuredangle ABD > \measuredangle ACB$

$\measuredangle ABC \gg \measuredangle ACB$

QEB.

Proposition 19

In any triangle the greater angle is subtended by the greater side

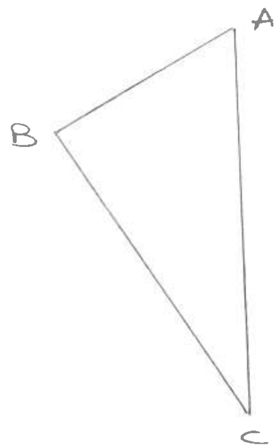
Given

$\triangle ABC$

$\measuredangle ABC > \measuredangle BCA$

Required:

$AC > AB$



If not:

$AC \leq AB$

But:

$AC \neq AB$

If not $\measuredangle ABC = \measuredangle ACB$ [1.5]

$AC \neq AB$

AC is not $< AB$

if not $\measuredangle ABC < \measuredangle ACB$ [1.18]

AC is not less than AB

$AC > AB$

Q.E.D.

Proposition 20

In any triangle two sides taken together in any manner are greater than the remaining one.

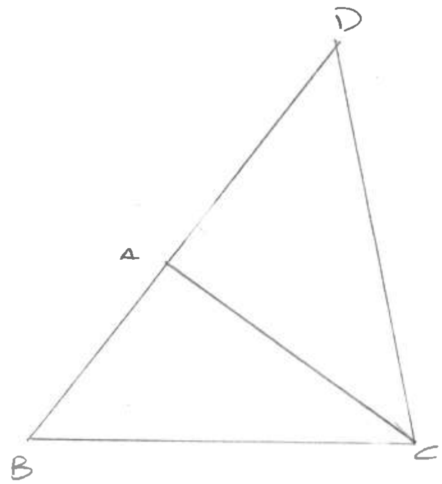
Given

ΔABC

Required

2 sides of ΔABC are greater than the remaining one

$BA, AC > BC$
$AB, BC > AC$
$BC, CA > AB$



$\overline{DA}$

$DA = AC$ [1.3]

$\overline{DC}$

$\measuredangle ADC = \measuredangle ACD$ [1.5]

$\measuredangle BCD > \measuredangle ADC$ [C.N. 5]

$\measuredangle BCD > \measuredangle BDC$

$DB > BC$ [1.19]

$DA = AC$

$BA, AC > BC$

$\therefore$

similarly

$AB, BC > AC$
$BC, CA > AB$

Q.E.D

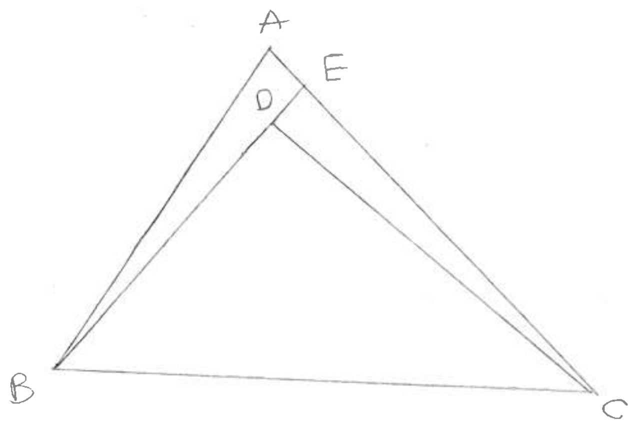
Proposition 21

If on one of the sides of a triangle, from its extremities, there be constructed two straight lines meeting within the triangle, the straight lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater angle.

Given:

$\triangle ABC$

BD, DC meeting inside the triangle.



required

BD, DC < BA, AC

BDC > BAC

$\overline{DE}$

(In Triangle ABE)

AB, AE > BE [I.20]

add EC

BA, AC > BE, EC

(In Triangle CED)

CE, ED > CD [I.20]

add DB

BE, EC > CD, DB

BA, AC >> BE, EC

∡BDC > ∡CED [I.16]

(In triangle ABE)

∡CEB > ∡BAC

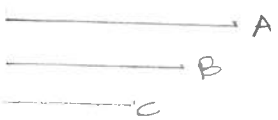
∡BDC >> ∡BAC

Q.E.D.

Proposition 22

Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one. [I.20]

Given.



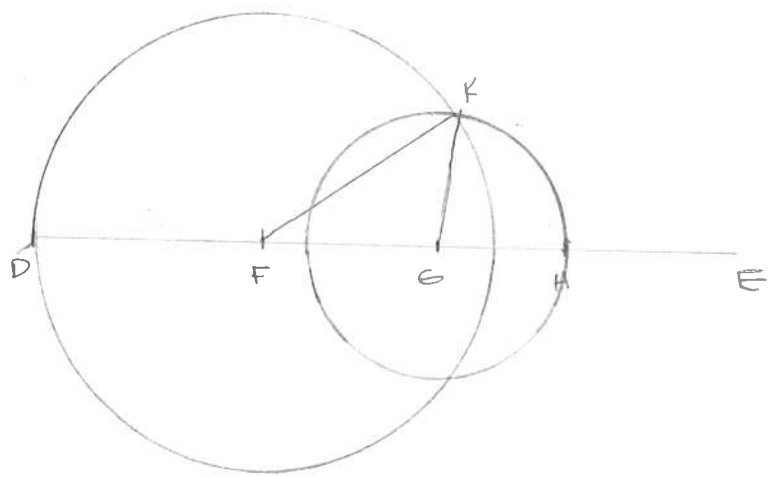
AB > C

AC > B

BC > A

required:

construct a triangle out of straight lines equal to A, B, C



$\overline{DE}$

DF = A

FG = B [I.3]

GH = C

⊙ DKL

⊙ KLH

FD = FK

FD = A

FK = A

GH = GK

GH = C

GK = C

FG = B

QEF

Proposition 23

On a given straight line and at a point on it to construct a rectilineal angle equal to a given rectilineal angle.

Given

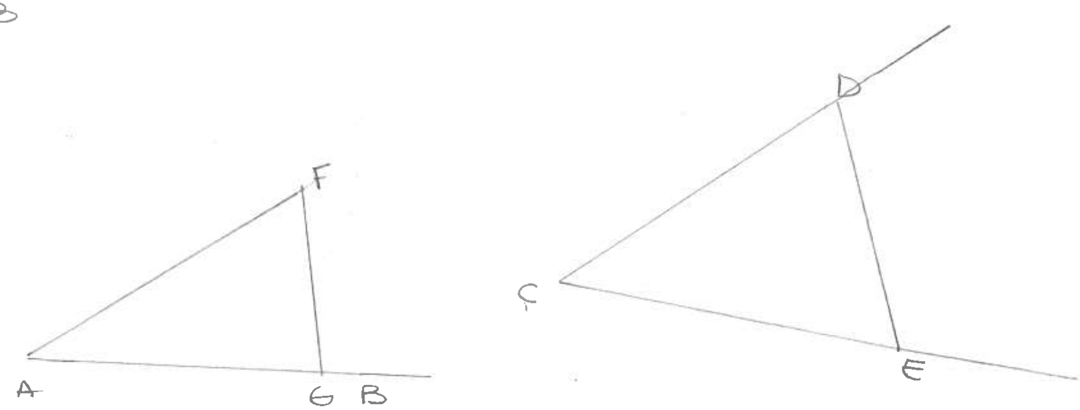
$\overline{AB}$

$\measuredangle DCE$ (given rectilineal angle)

required

to construct on AB

$\measuredangle = \measuredangle DCE$



DE (points taken at random

$\measuredangle AFG$

$CD = AF$

$CE = AG$ [I.22]

$DE = FG$

Since $DC = FA$

$CE = AG$ [I.8]

Then $DE = FG$

$\measuredangle DCE = \measuredangle AFG$

Proposition 24

If two triangles have the two sides equal to two sides respectively, but have one of the angles contained by the equal straight lines greater than the other, they will also have the base greater than the base

Given:

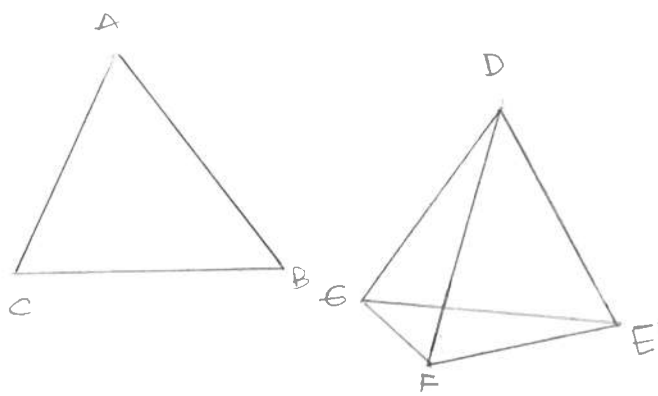
$\triangle ABC$

$\triangle DEF$

$AB = DE$

$AC = DF$

$\measuredangle A > \measuredangle D$



Required:

$BC > EF$

$\measuredangle EDG = \measuredangle BAC$ [I. 23]

$DG = AC = DF$

$\overline{EG}, FG$ [post. 1]

Since $BA = DE$, $AC = DG$, $\measuredangle BAC = \measuredangle EDG$ [I. 4]

Then $BC = EG$

Since $DF = DG$ then $\measuredangle DFG = \measuredangle DGF$ [I. 5]

$\measuredangle DFG > \measuredangle EGF$ [C N 5]

$\measuredangle EFG >> \measuredangle EGF$

$EG > EF$ [I. 19]

$EG = BC$

$BC > EF$

QED.

Proposition 25

If two triangles have two sides equal to two sides respectively, but have the base greater than the base, They will also have the one of the angles contained by the equal straight lines greater than the other.

given:

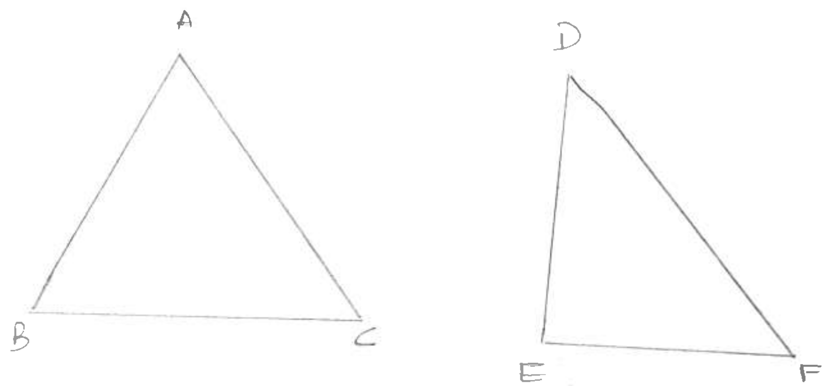
$\triangle ABC$
$\triangle DEF$

$AB = DE$
$AC = DF$

$BC > EF$

required:

$\measuredangle BAC > \measuredangle EDF$



$\measuredangle BAC \neq \measuredangle EDF$
↳ $BC \neq EF$ [I.4]

$\measuredangle BAC$ is not $< \measuredangle EDF$
↳ $BC < EF$ [I.24]

$\measuredangle BAC > \measuredangle EDF$

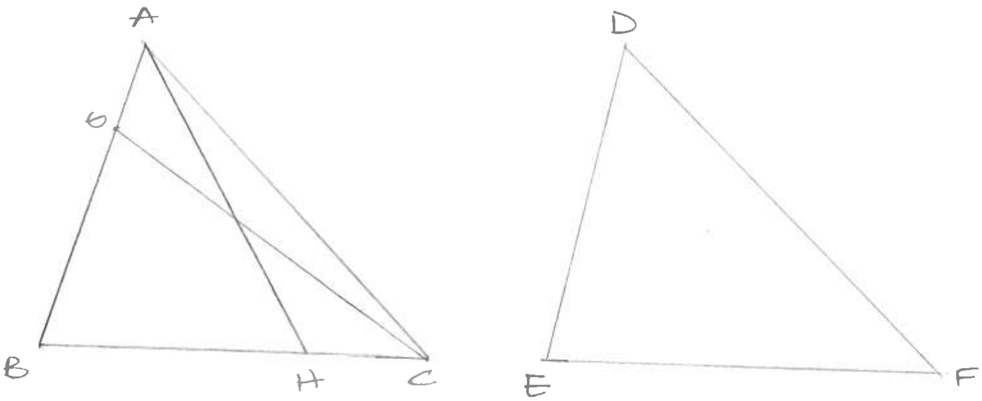
Q.E.D.

Proposition 26

If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle equal to the remaining angle.

Given.

$\triangle ABC$
$\triangle DEF$
$\measuredangle ABC = \measuredangle DEF$
$\measuredangle BCA = \measuredangle EFD$
$BC = EF$



equired

$AB = DE$
$AC = DF$
$\measuredangle BAC = \measuredangle EDF$

suppose:
$AB > DE$
$BG = DE$
GC
since $BG = DE$
$BC = EF$
Then $\measuredangle GBC = \measuredangle DEF$ [I.4]
$GC = DF$
$\triangle GBC = \triangle DEF$

$\measuredangle DFE = \measuredangle BCA$
$\measuredangle BCG = \measuredangle BCA$
$\rightarrow$ impossible

$AB = DE$

$\rightarrow BC = EF$
$AB = DE$

$\measuredangle ABC = \measuredangle DEF$ [I 4]
$AC = DF$
$\measuredangle BAC = \measuredangle EDF$

Given: $AB = DE$
required: $AC = DF$
$BC = EF$
$\measuredangle BAC = \measuredangle EDF$

suppose:
$BC > EF$
$BH = EF$
AH

since $BH = EF$
$AB = DE$
Then $AH = DF$ [I.4]
$\triangle ABH = \triangle DEF$
$\measuredangle BHA = \measuredangle EFD$

$BC = EF$

$\rightarrow AB = DE$
$BC = EF$

$AC = DF$

$\triangle ABC = \triangle DEF$

$\measuredangle BAC = \measuredangle EDF$ [I.4]

Q.E.D.

Proposition 27

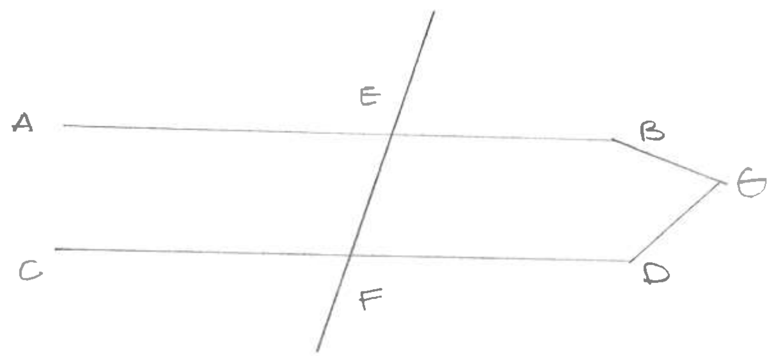
If a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another.

Given:

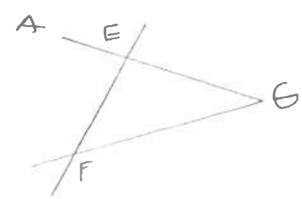
$\measuredangle AEF = \measuredangle EFD$

required:

AB ∥ CD



* imagine the extension like this.



* If not parallel, when AB, CD are produced, They will meet.

Produce B, D at G

→ in $\triangle GEF$

$\measuredangle AEF = \measuredangle EFG$ [I.16]
Impossible

Then when B, D are produced, They will not meet.

Straight lines which do not meet in either direction are parallel. [def. 23]

↳ AB ∥ CD

Q.E.D.

Proposition 28

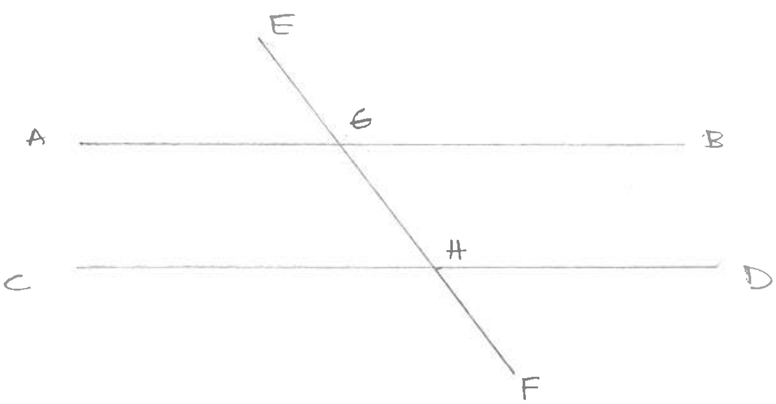
If a straight line falling on two straight lines make the exterior angle equal to the interior and opposite on the same side, or the interior angles on the same side equal to two right angles, the straight lines will be parallel to one another

Given:

$\measuredangle EGB = \measuredangle GHD$
or
$\measuredangle BGH = \measuredangle GHD$

Required:

$AB \parallel CD$



$\measuredangle EGB = \measuredangle GHD$
$\measuredangle EGB = \measuredangle AGH$ [I.15]
$\measuredangle AGH = \measuredangle GHD$
They are alternate [I.27]

↳ $AB \parallel CD$

$\measuredangle BGH, \measuredangle GHD = 2L$
$\measuredangle AGH, \measuredangle BGH = 2L$ [I.13]

$\measuredangle AGH, \measuredangle BGH = \measuredangle BGH, \measuredangle GHD$

- subtract $\measuredangle BGH$

$\measuredangle AGH = \measuredangle GHD$

And they are alternate

↳ $AB \parallel CD$ [I.27]

Proposition 29

A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.

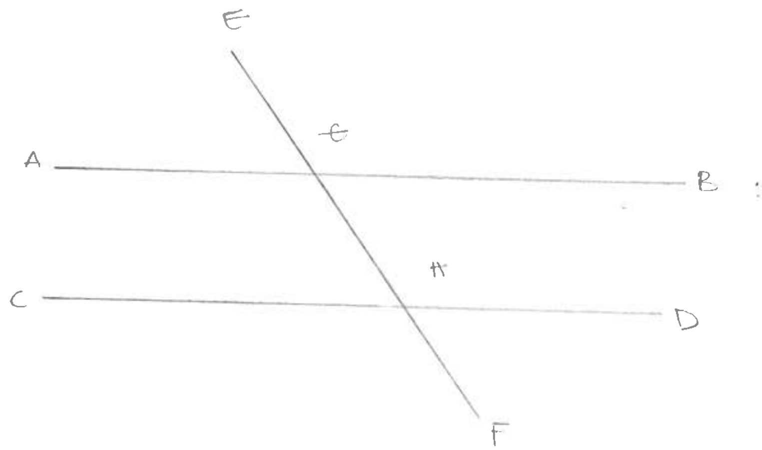
Given:

required:

$\measuredangle AGH = \measuredangle GHD$

$\measuredangle EGB = \measuredangle GHD$

$\measuredangle BGH, \measuredangle GHD = 2L$



Suppose:

$\measuredangle AGH$ is not equal to $\measuredangle GHD$

$\measuredangle AGH > \measuredangle GHD$

add $\measuredangle BGH$

$\measuredangle AGH, \measuredangle BGH > \measuredangle BGH, \measuredangle GHD$

$\measuredangle AGH, \measuredangle BGH = 2L$ [I.13]

$\measuredangle BGH, \measuredangle GHD < 2L$

But $AB \parallel CD$

$\measuredangle AGH = \measuredangle GHD$

$\measuredangle AGH = \measuredangle EGB$ [I.15]

$\measuredangle EGB = \measuredangle GHD$ [C.N.1]

add $\measuredangle BGH$

$\measuredangle EGB, \measuredangle BGH = \measuredangle GHD, \measuredangle BGH$ [C.N.2]

$\measuredangle EGB, \measuredangle BGH = 2L$ [I.13]

$\measuredangle BGH, \measuredangle GHD = 2L$

Q.E.D

Proposition 30

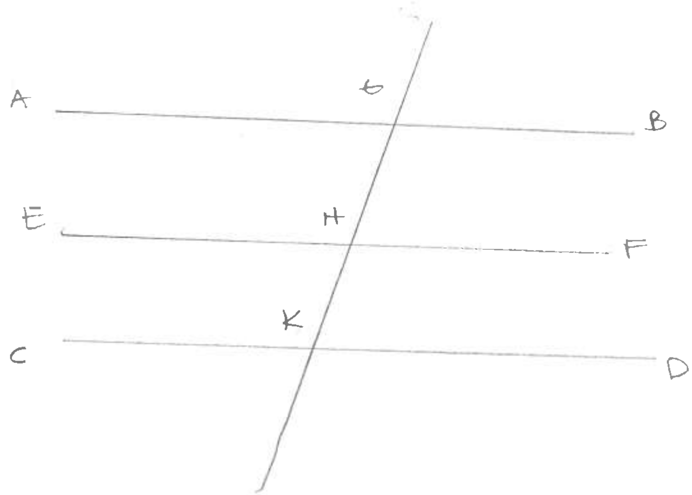
Straight lines parallel to the same straight line are also Parallel to one another.

Given:

$AB, CD \parallel EF$.

required:

$AB \parallel CD$



$\overline{GK}$

$\angle AGK = \angle GHF$ [I.29]

$\angle GHF = \angle GKD$ [I.29]

$\angle AGK = \angle GKD$ [C.N. 1]

$AB \parallel CD$

Q.E.D.

Proposition 31

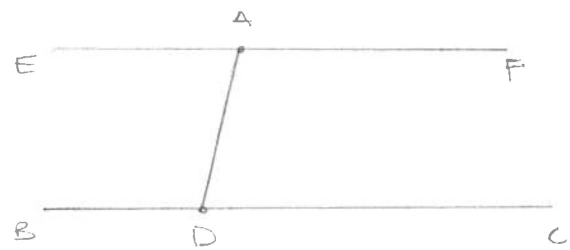
Through a given point draw a straight line parallel to a given straight line

Given

$\overline{BC}$ $\cdot A$

required:

Through $\cdot A$
$EF \parallel BC$



$\cdot D$ (at random)

$\overline{AD}$

$\measuredangle DAE = \measuredangle ADC$ [I.23]

$\overline{AF}$ [Post. 2]

$\measuredangle EAD = \measuredangle ADC$

$EAF \parallel BC$

Q.E.F

Proposition 32

In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles.

Given:

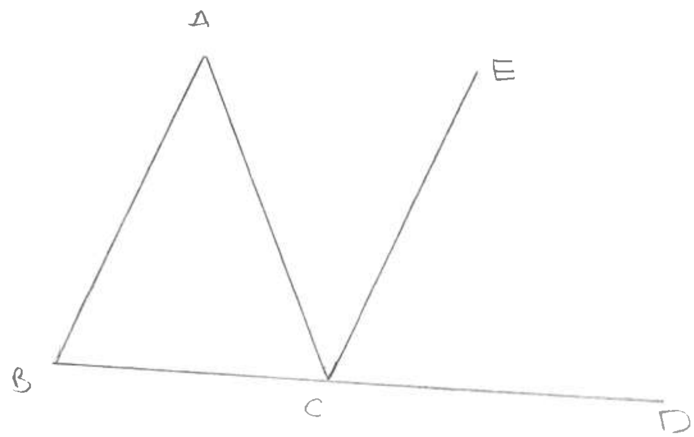
$\triangle ABC$

$BC \rightarrow D$

required:

$\measuredangle ACD = \measuredangle CAB, \measuredangle ABC$

$\measuredangle ABC, \measuredangle BCA, \measuredangle CAB = 2\llcorner$



$CE \parallel AB$ [I.31]

$\measuredangle BAC = \measuredangle ACE$ [I.29]

$\measuredangle ECD = \measuredangle ABC$ [I.29]

$\measuredangle ACD = \measuredangle ABC, \measuredangle BAC$.

-add ACB

$\measuredangle ACD, \measuredangle ACB = \measuredangle ABC, \measuredangle BAC, \measuredangle ACB$

$\measuredangle ACD, \measuredangle ACB = 2\llcorner$ [I.13]

$\measuredangle ABC, \measuredangle BAC, \measuredangle ACB = 2\llcorner$

Q.E.D.

Proposition 33

The straight lines joining equal and parallel straight lines [at the extremities which are] in the same directions [respectively] are themselves also equal and parallel.

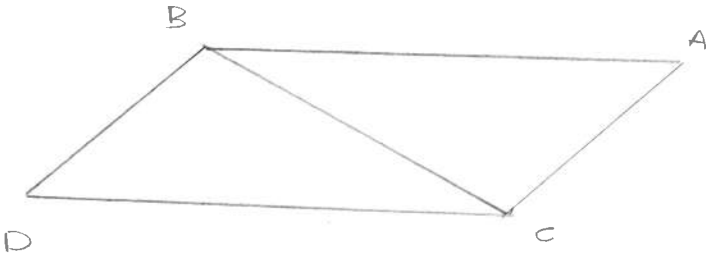
Given:

$AB = \parallel CD$

AC, BD.

required:

$AC \underset{=}{\parallel} BD$



$\overline{BC}$

$\measuredangle BCD = \measuredangle ABC$ [I.29]

$AB, BC = BC, CD$

$\measuredangle ABC = \measuredangle BCD$

$AC = BD$

$\triangle ABC = \triangle BCD$ [I.4]

$\measuredangle ACB = \measuredangle CBD$

$AC \parallel BD$ [I.27]

$AC = BD$

Q.E.D

Proposition 34

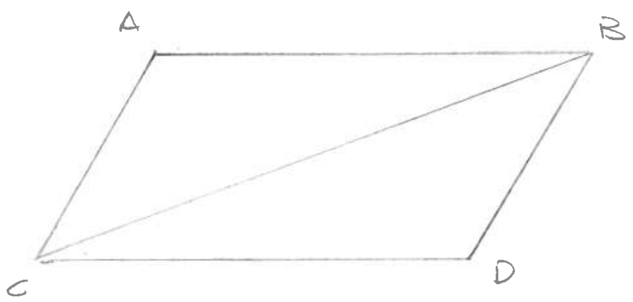
In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.

Given:

ACDB parallelogramic area
BC diameter

required:

opposite sides and angles are equal
BC bisects ACDB



$\measuredangle ABC = \measuredangle BCD$ [I.29]

$\measuredangle ACB = \measuredangle CBD$ [I.29]

ΔABC, $\measuredangle ACB = \measuredangle BCD$, $\measuredangle CBD$
BC common.

$\hookrightarrow AB = CD$
$AC = BD$
$\measuredangle CAB = \measuredangle BDC$ [I.26]

$\measuredangle ABD = \measuredangle ACD$ [C.N.2]
$\measuredangle CAB = \measuredangle BDC$

$AB = CD$ & BC is common.

$AB, BC = DC, CB$
$\measuredangle ABC = \measuredangle BCD$
$\Delta ABC = \Delta DCB$ [I.4]

So BC bisects the Parallelogram

Q.E.D

Proposition 35

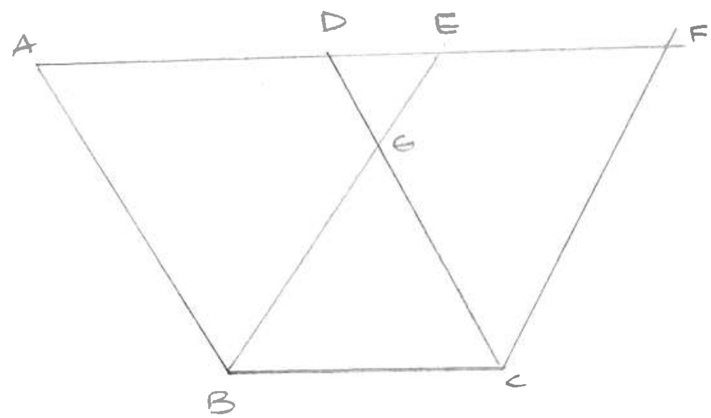
Parallelograms which are on the same base and in the same Parallels are equal to one another.

Givens:

ABCD ▱
▱ EBCF
Betn on the base BC and in AF, BC

required

▱ ABCD = ▱ EBCF



AD = BC [I.34]
EF = BC [I.34]
AD = EF [C.N. 1]

DE common
AE = DF [C.N. 2]

AB = DC [I.34]

EA, AB = FD, DC
∡FDC = ∡EAB [I.29]
EB = FC
△EAB = △FDC [I.4]

→ Subtract △DGE from each

ABDG = FCGE [C.N. 3]
→ add △GBC
▱ ADBC = ▱ EFBC [C.N. 2]

Q.E.D.

Proposition 36

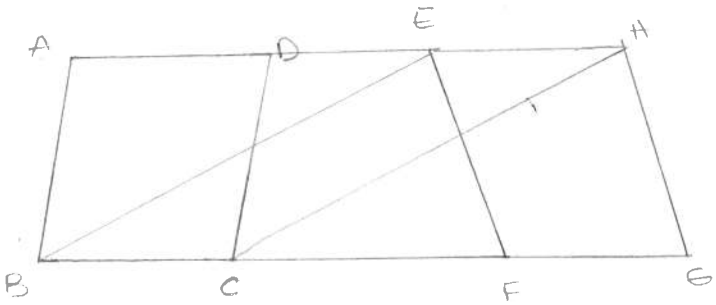
Parallelograms which are on equal bases and in the same parallels are equal to one another.

Given:

▱ ABCD
▱ EFGH
re on bases BC, FG
nd on parallels AH, BG

equired:

▱ ABCD = ▱ EFGH



$\overline{BE}$
$\overline{CH}$

BC = FG
FG = EH
BC = EH [C.N 1]
BC ∥ EH
EB ∥ HC [I.33]
▱ EHBC [I.34]
▱ EHBC = ▱ ABCD
↳ BC common
and same ∥ BC, AH [I.35]

For the same
▱ EFGH = ▱ EBCH [I.35]
▱ EFGH = ▱ ABCD [C.N 1]

QED.

Proposition 37

Triangles which are on the same base and in the same parallels are equal to one another.

given:

$\triangle ABC$

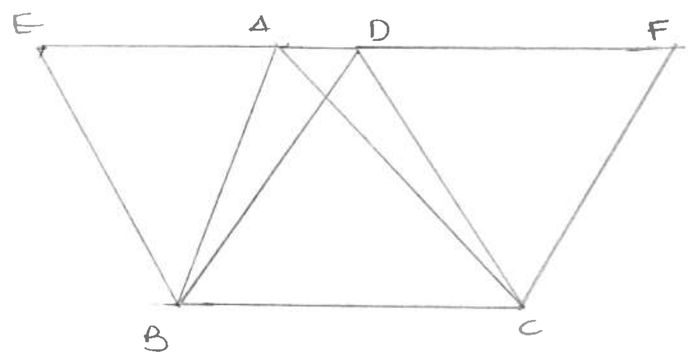
$\triangle DBC$

same base BC

same parallels AD, BC

required:

$\triangle ABC = \triangle DBC$



AD → E, F

BE ∥ CA [I.31]

CF ∥ BD [I.31]

▱ EBCA

▱ DBCF

▱ EBCA = ▱ DBCF [I.35]

AB intersects ▱ EBCA [I.34]

↳ $\triangle ABC$ is half of ▱ EBCA

DC intersects ▱ DBCF [I.34]

↳ $\triangle DBC$ is half of ▱ DBCF

Halves of equal things equal one another

$\triangle ABC = \triangle DBC$

Q.E.D

Proposition 38

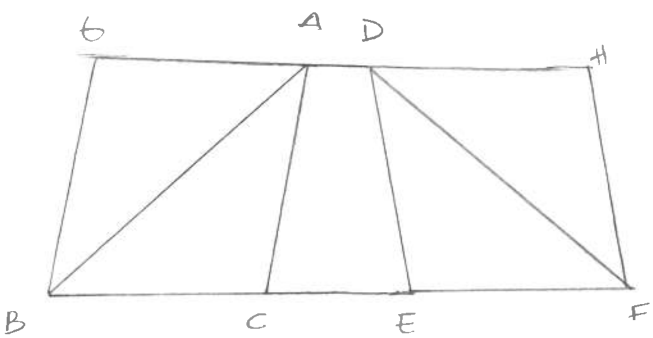
Triangles which are on equal bases and in the same parallels are equal to one another.

given:

△ABC
△DEF
BC = EF (Base)
same parallels BF, AD

required:

△ABC = △DEF



AD → GH

BG ∥ CA [I.31]
FH ∥ DE [I.31]

▱GBCA
▱DEFH
Same base BC, EF
same parallels BF, GH [I.36]
▱GBCA = ▱DEFH

AB bisects ▱GBCA
△ABC is half of ▱GBCA [I.34]
DF bisects ▱DEFH
△DEF is half of ▱DEFH [I.34]

halves of equal things are equal to one another
△ABC = △DEF

Q.E.D.

Proposition 39

Equal triangles which are on the same base and on the same side are also in the same parallels.

Given:

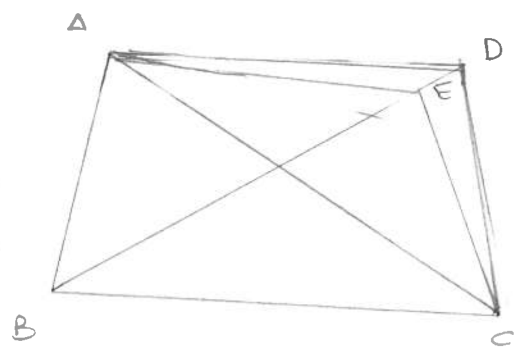
$\triangle ABC = \triangle DBC$

Same base BC

required:

Same parallels

AB ∥ DC



$\overline{AD}$

suppose:

AE ∥ BC [I.31]

$\overline{EC}$

$\triangle ABC = \triangle EBC$ [I.37]

same base and parallels

But... $\triangle ABC = \triangle DBC$

$\triangle DBC = \triangle EBC$

impossible!

AE is not parallel to BC

AD ∥ BC

Q.E.D.

Proposition 40

Equal triangles which are on equal bases and on the same side are also in the same parallels.

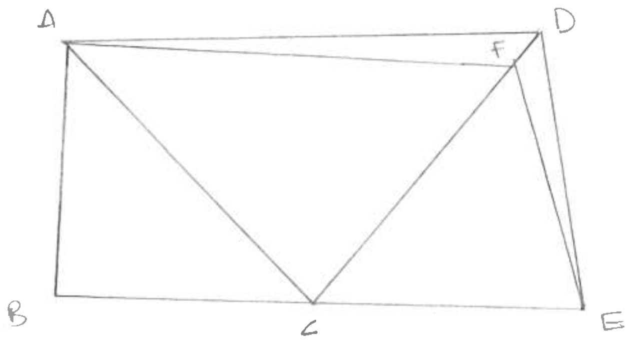
given:

$\triangle ABC = \triangle CDE$

$BC = CE$

required

Both triangles are on the same parallels.



$\overline{AD}$

suppose

$AF \parallel BE$

$\overline{FE}$

$\triangle ABC = \triangle FCE$

For they are in the same base & same parallels [I.38]

$\triangle ABC = \triangle CDE$

$\triangle FCE = \triangle CDE$ [C.N. 1]

impossible

AF is not $\parallel$ to BE

$AD \parallel BE$

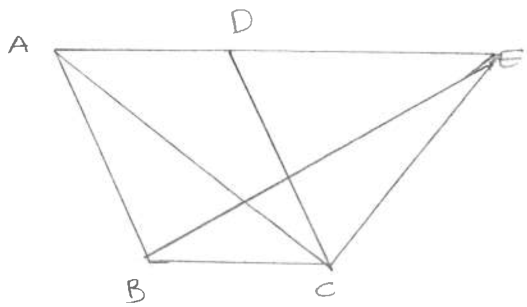
Q.E.D.

Proposition 41

If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle.

given:

▱ABCD & △EBC
have the same base
BC and are in the
same parallels BC, AE



required:

▱ABCD is double
△EBC.

$\overline{AC}$

△ABC = △EBC
same base BC &
same parallels BC, AE [I.37]

AC bisects ▱ABCD [I.34]
▱ABCD is double △ABC

▱ABCD is double △EBC

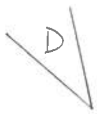
Q.E.D.

Proposition 42

To construct, in a given rectilineal angle, a parallelogram equal to given triangle.

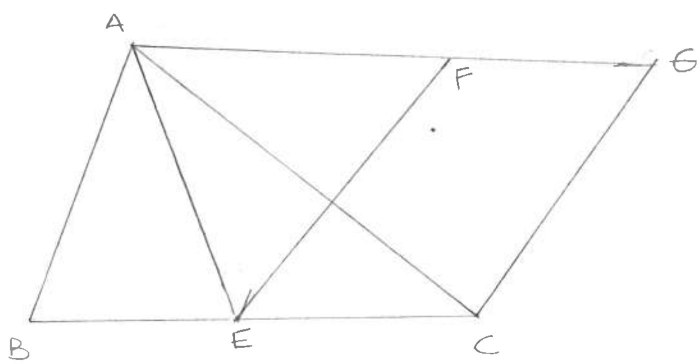
given:

$\triangle ABC$
$\measuredangle D$



required:

in $\measuredangle D$ a
▱ = $\triangle ABC$



Bisect BC at E
$\overline{AE}$
$\measuredangle CEF = \measuredangle D$ [I.23]
AG ∥ EC
CG ∥ EF [I.31]
▱ FECG
BE = EC
ABE = AEC
They are on equal bases [I.38]
and same parallels
$\triangle ABC$ is double $\triangle AEC$
▱ FEGC is double $\triangle AEC$
▱ FECG = $\triangle ABC$
$\measuredangle CEF = \measuredangle D$

Q.E.F.

Proposition 43

In any parallelogram the complements of the parallelograms about the diameter are equal to one another

given

▱ABCD

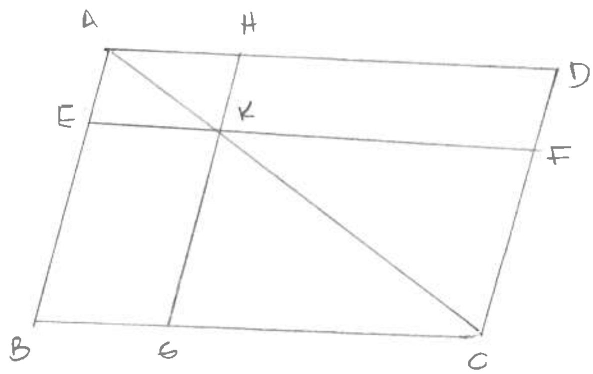
AC diameter

EH, FG ▱

BK, KD complements

required:

complement BK = KD



△ABC = △ACD [I.34]

▱EH

AK is diameter

△AEK = △AKH

▱FG

KC is diameter

△KFC = △KGC

△AEK, △KGC = △AHK, △KFC [C.N.2]

△ABC = △ACD

↳ complements BK = KD [C.N.3]

QED

Proposition 44

To a given straight line to apply, in a given rectilineal angle, a Parallelogram equal to a given triangle.

given:

$\overline{AB}$ $\angle D$

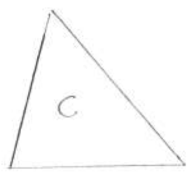
$\triangle C$

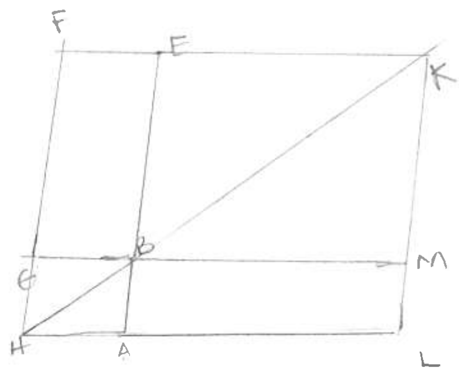
required:

on $\overline{AB}$ apply

$= \angle D$

$= \triangle C$





▱BEFG = $\triangle C$
in angle EBG = D [I.42]

FG → H

AH ∥ BG, FE [I.31]

$\overline{HB}$

$\angle$AHF, $\angle$HFE = 2L
HF falls on parallels FE, HA [I.29]

$\angle$BHG, $\angle$GFE < 2L
IF produced indefinitely they will meet. [POST.5]

Produce HB, FE & let them meet at K

KL ∥ EA, FH [I.31]

Produce HA, GB to L, M

▱HLKF
Diameter HK

▱AG
▱ME
LB, BF Their complements about HK
LB = BF [I.43]

BF = C
LB = C [C.N. 1]

Q.E.F

Proposition 45

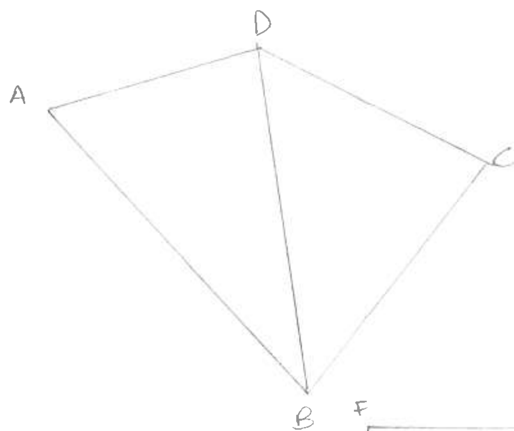
To construct, in a given rectilineal angle, a parallelogram equal to a given rectilineal figure.

given:

$\diamond$ ABCD
$\angle$E

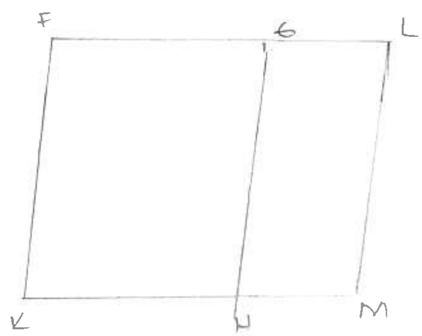
required

construction
$\angle$E a ▱ = ABCD



$\overline{DB}$
▱FH = $\triangle$ABD
$\angle$HKF = E [I.42]

▱GM = $\triangle$DBC
$\angle$GHM = E [I.44]



$\angle$GHM = $\angle$HKF [C.N. 1]
→ add $\angle$KHG
$\angle$GHM, $\angle$KHG = $\angle$HKF, $\angle$KHG

$\angle$HKF, $\angle$KHG = 2b [I.29]
$\angle$GHM, $\angle$KHG = 2b

KH is in a straight line with HM [I.14]
because their adjacent angles = 2b

$\angle$MHG = $\angle$HGF
HG falls on the same parallels FL, KM. [I.29]

→ add $\angle$HGL
$\angle$MHG, $\angle$HGL = $\angle$HGF, $\angle$HGL. [C.N. 2]

$\angle$HGF, $\angle$HGL = 2b
$\angle$MGH, $\angle$HGL = 2b [C.N. 1]

FK ∥ HG [I 34]
HG ∥ LM
KF ∥ ML [C.N. 1][I.30]
KM, LF join them at their extremities
KM ∥ FL [I.33]

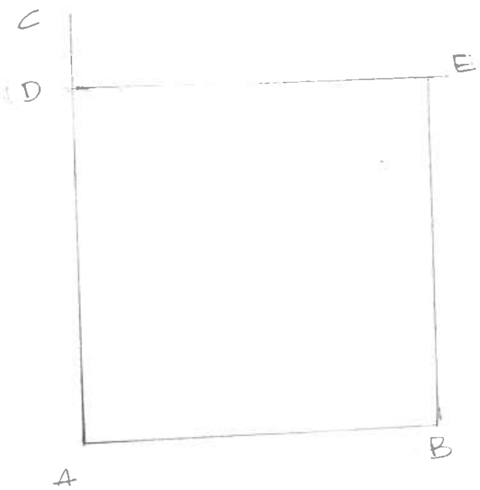
▱KLMF

$\triangle$ABD = ▱FH
$\triangle$DBC = ▱GM
ABCD = ▱KFLM

QEF

Proposition 46

on a given straight line describe a square.



ven:

B

quired:

I
c right angled

E
at right angle [I.11]
with $\overline{AB}$

D = AB
DE ∥ AB [I.31]
B ∥ DA

▱ ADEB
AB = DE [I.34]
BE = AD

B = AD
AB = AD = DE = BE
▱ ADEB is equilateral

∡BAD, ∡ADE = 2L [I.29]
AD falls on parallels AB, DE

opposite angles ∡ABE, ∡DEB are right
* In parallelogramic areas the opposite sides and angles are equal to one another [I.34]

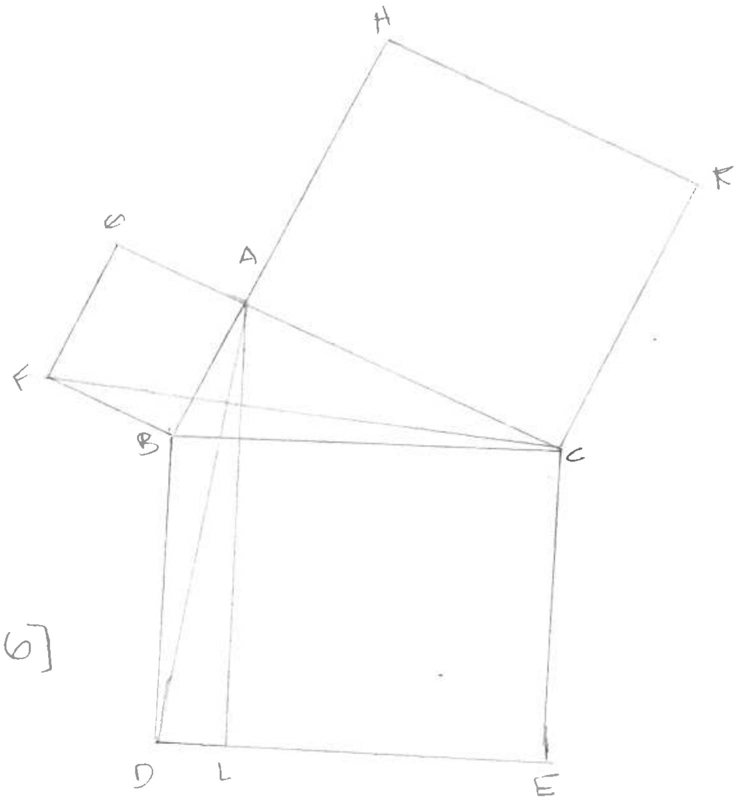
ABDE is right angled
and
equilateral

Therefore, a square

Q.E.F.

Proposition 47

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.



given:

△ABC
∡BAC right

required:

BC = to the squares on BA, AC

□BDEC on BC
□GB on BA [I.46]
□AC on HC

AL ∥ BD

$\overline{AD}$

$\overline{FC}$

∡BAC, ∡BAG = 2L
CA is in straight line with AG [I.14]
∡BAC, ∡HAG = 2L
BA is in straight line with AH [I.14]

∡DBC = right
∡FBA = right
∡DBC = ∡FBA
→ add ∡ABC
∡DBC, ∡ABC = ∡FBA, ∡ABC
∡DBA = ∡FBC [C.N. 2]
Base AD = base FC
△ABD = △FBC [I.4]

▱BL is double △ABD
same base BD & same parallels BD, AL [I.41]
▱GB is double △FBC
same base BC & same parallels FB, GC [I.41]
□BL = □GB

similarly:

AE, BK joined
□CL = □HC
□BDEC = □GB, □HC [C.N. 2]

□BC = □BA, □AC.

Q.E.D.

Proposition 48

If in a triangle the square on one of the sides be equal to the square on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.

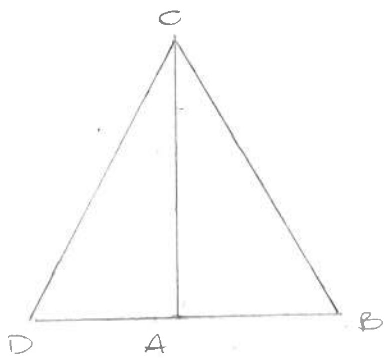
given:

ΔABC

□BC = □BA, □AC

equired:

∡BAC is right.



AD → A

AD = AB

DC

□DA = □AB
because AD = AB

add AC

□DA, AC = □AB, AC

□DC = □DA, □AC
since ∡DAC is right [I.47]

□BC = □BA, □AC

□DC = □BC

DC = BC

DA = AB
AC common

DA, AC = BA, AC
Base DC = BC

∡DAC = ∡BAC [I.8]

∡DAC is right

∡BAC is right.

Q.E.D